2010 American Control Conference
Marriott Waterfront, Baltimore, MD, USA
June 30-July 02, 2010

Applying Optimal Control Using SLP on a Hydraulic System

Christoph Verlohren
Institute of Automatic Control
Laboratory for Control Engineering and Mechatronics
64283 Darmstadt, Germany
c.verlohren@gmx.de

Marco Muenchhof
Institute of Automatic Control
Laboratory for Control Engineering and Mechatronics
64283 Darmstadt, Germany
mmuenchhof@iat.tu-darmstadt.de

Tarunraj Singh
Department of Mechanical & Aerospace Engineering
University at Buffalo
Buffalo, NY-14260
tsingh@buffalo.edu

Abstract— **This paper applies an optimal control approach to an hydraulic system. An approach is proposed to use an open-loop control on the model of a real hydraulic system. The approach uses a feedback control, to create a closed-loop control. The results of this approach are tested against different linear controls applied to the hydraulic system. The paper shows that the proposed approach can be used on oscillatory and also on systems, where the system dynamics are not exactly known.**

I. Introduction

In the past different algorithms have been developed to design optimal control inputs for nonlinear systems. The COPS 3.0 paper [6] from the Argonne National Laboratory gives an overview over some of these algorithms. Singh and Singla [10] have introduced an algorithm using a Sequential Linear Programming (SLP) approach to calculate optimal control profiles. Singh et al. [9] and Verlohren [11] have introduced an extension to this approach. All those algorithms only have been tested on theoretic systems. This paper will extend the prior test and apply the SLP approach to a verified model of a nonlinear-hydraulic system. Since the SLP Algorithm creates an open-loop control, an adjustment is going to be applied to create a closed-loop control. This will allow to apply the open-loop control even if the system specifications have been changed. These results will be compared with a linear control design discussed and applied in the thesis of Janda [3].

II. Basics

The following chapter will give an introduction into the hydraulic system and its properties. The control design done by Janda [3] will be shortly discussed. Furthermore this section will give a brief introduction into the Time Optimal Control design using the SLP algorithm discussed in [10], [9] and [11]. In the end of this chapter the idea behind the transformation from a feed-forward control to a closed-loop control will be introduced. In sections II-B and II-C the control designs will be discussed shortly.

A. Hydraulic System

Hydraulic systems are widely used in the industry. Their advantage is that they can be used to apply large forces. As examples industrial robots or presses can be named. The here observed system will be used as an example for such system. This section will use the dissertation from Münchhof [7] as basis to describe the analyzed hydraulic system.

A *swash plate axial piston pump* is used to suck the hydraulic oil out of the storage tank. The oil is then pumped through *supply lines* and connected to a *proportional valve* and a *differential cylinder* which act as a servo axis. As a *mechanical load* a spring supported by a carriage is used. The valve spool position of the *proportional valve* is used as the input and the position of the *mechanical load* is the measured output.

Figure 1 shows a more mathematical scheme of the hydraulic system. In the top left of Fig. 1 the numerical control input y_{Com} is shown. The mechanical load is shown on the right side of Fig. 1. Münchhof [7] determined and verified the model given in equations (1)-(3).

$$(V_{0A} + A_A y(t)) \frac{1}{E(p_A,T)} \dot{p}_A(t) + A_A \dot{y}(t) = \dots$$
$$\dots \left(\dot{V}_A(p_A,p_P,T,y_v) - \dot{V}_{AB}(p_A,p_B,T) \right) \quad (1)$$

$$\dot{V}_{AB}(p_A,p_B,T) = \alpha_D A \sqrt{\frac{2}{p(p,T)}} \cdot \dots$$
$$\dots \cdot \sqrt{|p_A(t) - p_B(t)|}\,\mathrm{sign}(p_A(t) - p_B(t)) \quad (2)$$

$$\dot{V}_{AB}(p_A,p_B,T) = G_{AB}(T)(p_A(t) - p_B(t)) \quad (3)$$

B. Linear Control

The standard controls in practical applications are typically linear controls, such as P-,PI- or PID-controls. This section will give a brief overview of the work done by Janda [3]. One goal of this thesis was to create optimal linear P- and PI-controls for the nonlinear hydraulic system. To determine

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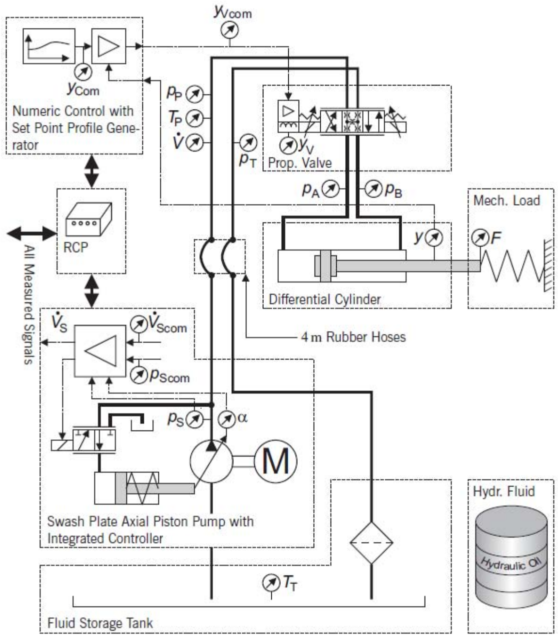

Fig. 1. Scheme of the hydraulic system [7]

TABLE I
PERFORMANCE INDICES [1]

Name	Definition	Overshoot	Transient Time
IAE	$\int \lvert e(t)\rvert \mathrm{d}t$	medium	medium
ITAE	$\int \lvert e(t)\rvert t\mathrm{d}t$	small	large
ISE	$\int e^2(t)\mathrm{d}t$	large	small

the optimality, different performance indices, listed in Table I were being used.

As an approach to establish optimal P- and PI-controls a series of simulations based on the nonlinear system were carried out with varying control parameters. Based on a performance index, here the IAE was chosen, the control parameters were determined. For a P-control this is a 1-D search. For a PI-control this is, because of the two control parameters, a 2-D search. In section III-A this approach will be used to determine the required control parameters.

C. Optimal Control

Optimal control has the objective to find a control profile which causes a system to satisfy certain constraints and maximize or minimize a certain performance index. For example one might want to find an optimal control to move a nonlinear two mass spring system from one state into another as fast as possible.[8]

This paper only deals with time optimal control problems. The considered performance index is given by

$$J = \int_{t_0}^{t_f} dt = t_f - t_0. \tag{4}$$

A way to solve this problem using SLP will be shown in section II-D.

D. Time Optimal control using SLP

The SLP algorithm uses a bisection approach combined with an LP formulation to create an optimal control profile.

The system model which is consulted in this work is

$$\dot{x}(t) = f(x(t),u(t)) \tag{5}$$

$$y(t) = g(x(t),u(t)) \tag{6}$$

which represents the dynamic behavior of linear or nonlinear dynamic systems. The form of the linear programming problem (7) - (9) suggests that the system equations have to be transformed in such a way that they fit the required form. To do so the system has to be approximated as a linear discrete-time system, which will be done by using a stepwise linearization approach. The following section is a short summary of the paper from Singh and Singla [10], which introduces sequential linear programming (SLP) to solve optimal control problems for nonlinear systems.

1) Linear Programming: Linear programming (LP) is used to maximize (minimize) a given linear equation (7) under certain constraints. An LP is for example used to maximize production in a factory. In those cases the LP is used to maximize (minimize) the objective function, which represents costs and revenue of products subject to certain restrictions.

The general form of a LP problem can be written as

$$\max \underline{c}^T \ \underline{x} \tag{7}$$

$$\text{s.t. } \underline{A} \ \underline{x} \leq \underline{B} \tag{8}$$

$$\underline{A}_{eq} \ \underline{x} = \underline{B}_{eq}. \tag{9}$$

2) The Bisection Algorithm: The proposed algorithm only checks for feasibility of the LP problem. The actual optimization is done by the Bisection Algorithm. A lower bound t_f^L and an upper bound t_f^U for the cost function are guessed. The Bisection Algorithm will find the smallest value t_f for the cost function which yields a feasible solution for the LP problem, which fulfills the required tolerance.

3) Time Optimal Control: For the time optimal control the Bisection Algorithm follows the outline of Fig. 2. The starting value for t_f is guessed as the middle between the lower and the upper bound

$$t_f = \frac{t_f^L + t_f^L}{2}.$$

If the difference between the lower bound t_f^L and the upper bound t_f^U is smaller than a specified tolerance ϵ the system will be discretized with N samples over the time interval $[0 \quad t_f]$. Based on the new discrete system, the LP problem is stated. This process is described in [10]. This new LP problem will be solved under the given constraints. If the resulting LP problem is feasible, the upper bound is changed to

$$t_f^U = t_f. \tag{10}$$

If the problem is infeasible the lower bound will be changed to

$$t_f^L = t_f. \tag{11}$$

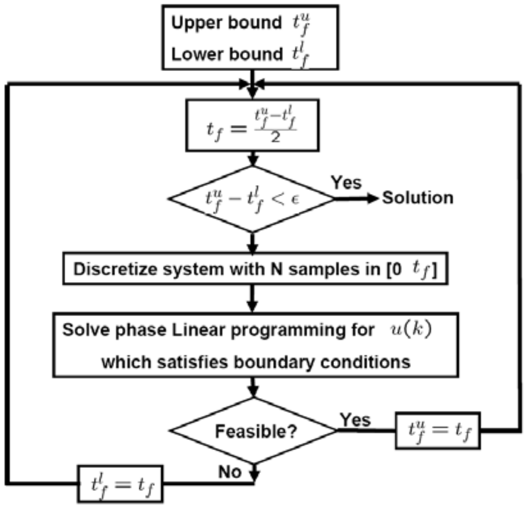

Fig. 2. Time optimal bisection algorithm [10]

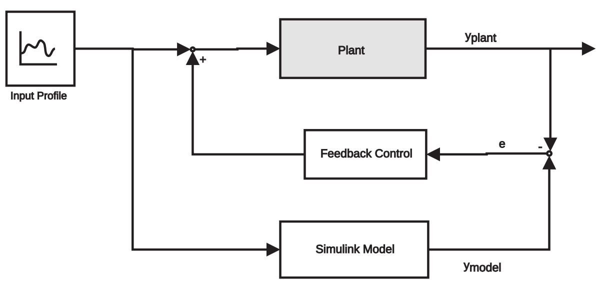

Fig. 3. Block diagram of feedback control transition

This process continues until either the tolerances are fulfilled, the maximum number of iterations are exceeded or the LP solver doesn't find a feasible solution. The papers by Singh et al. [9], [10] explain how the actual LP problem is stated.

E. From Open-Loop to Closed-Loop Control

An Optimal Control designed using the SLP algorithm results in an feedforward control. Because of the open-loop character of the control the actual behavior of the system is not taken into consideration. Especially if the model of the system is not exact or the parameters of the system have changed or vary an open-loop control will not take this into account. Therefore a transformation into a closed-loop control has been developed. The resulting block diagram is shown in Fig. 3. As an input signal the input profile designed with the SLP algorithm is used. This input signal is used to actuate the real system as well as a Simulink model, based on the assumed system dynamics and parameters. Because of the probable difference between the model and the real system a control is introduced which will compensate this difference.

This approach will be tested and discussed in section IV.

TABLE II

P-CONTROL: $K_{\mathrm{P,opt}}$ NUMERICAL OPTIMIZED CONTROL PARAMETER [3]

m [kg]	$K_{\mathrm{P,opt}}$ $\left[\frac{1}{m}\right]$
53	100
100	90
225	75
500	50
750	45
1000	40

TABLE III

PI-CONTROL: K_x NUMERICAL OPTIMIZED CONTROL PARAMETERS [3]

m [kg]	$K_{\mathrm{P,opt}}$ $\left[\frac{1}{m}\right]$	$K_{\mathrm{I,opt}}$ $\left[\frac{1}{ms}\right]$
53	170	2900
100	160	2800
225	120	2200
500	55	700
750	45	500
1000	40	400

Different simulations will be run with varying parameters. These results will then be compared to a standard linear control approach.

III. APPLIED CONTROL DESIGN

The following section will give a short overview over the design process of the different used controls. First it starts off by stating the results of the linear control. Then a more detailed description of the process of finding a time optimal control profile will be given and in the end of the chapter the process of designing the feedback control will be stated.

A. Applied Linear Control

As described in section II-B a P- and a PI-control will be designed for the nonlinear hydraulic system. To determine the control parameters a series of simulations has been done by Janda [3]. A brief overview over the results will be given at this point.

For the P-control a 1-D search has been used to determine the optimal control parameter, based on the IAE performance index. This search has been done for different mass loads. The resulting control parameters are given in Table II.

To determine the optimal control parameters for the PI-control a 2-D search has to be done. Again the search has been done with different mass loads. The 2-D search process is shown in Fig. 4. The figure displays the IAE performance index for a mass load of $53\mathrm{kg}$. The minimum of this functions determines the optimal control parameters for the PI-control. For different mass the resulting control parameters are displayed in Table III.

B. Applied Time Optimal Control

In contrast to the section above, the SLP algorithm does not only provide contoller parameters, but designs an actual input profile. This input profile can be used to transform the discussed system from one state into another. The paper assumes the initial state

$$x_0 = \left[0,\ 0,\ 0,\ 0.15,\ 0,\ 20\cdot 10^5,\ 20\cdot 10^5\right], \qquad (12)$$

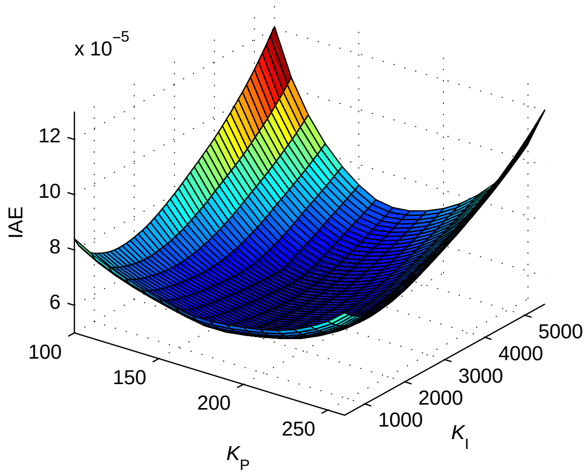

Fig. 4. Meshgrid of the PI-control parameters for $m = 53\,\mathrm{kg}$ [3]

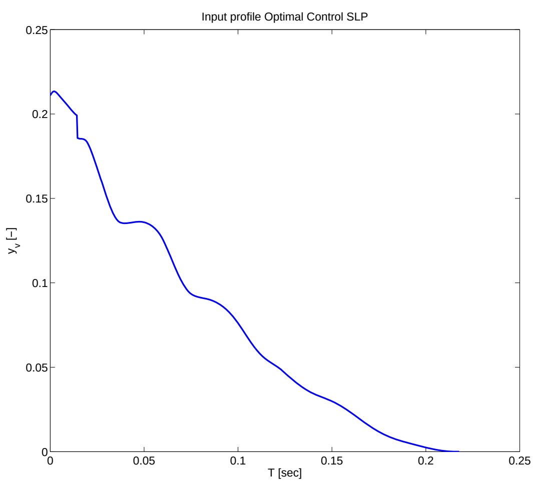

Fig. 5. Time optimal input profile

where

$$x = [y_v,\ \dot{y_v},\ \ddot{y_v},\ y_k,\ \dot{y}_k,\ p_A,\ p_B]\,.$$

For the final state only the position y_k and the velocity $\dot{y}_k$ is important, with $x_f = [y_{k,f},\ \dot{y}_{k,f}]$.

$$x_f = [0.15,\ 0] \tag{13}$$

As a mass for the hydraulic system $m = 500\mathrm{kg}$ is chosen.

After running the SLP algorithm with $N = 601$. N being the number of samples and the number of points of the input profile. As an upper bound for the optimal time $0.5sec$ is chosen. The algorithm returns the input profile shown in Figure 5.

Since the SLP algorithm is a numerical approach, the typical problems associated with numerical algorithms have occurred during the calculation. It was especially hard to find the right combination of an upper bound for the optimal time and a good starting profile as well as the scaling factor α.

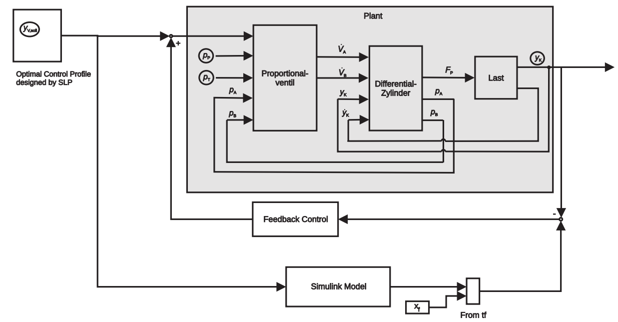

Fig. 6. Modified feedback block diagram

This problem has already been discussed in the thesis by Verlohren [11]. In contrast to earlier calculations this time a numerical model is solved by using a fixed step integration. The fixed step integration requires a large number of samples and the algorithm to calculate the equality matrices in the SLP can only be used with a fairly small number of samples, because otherwise the calculation time increases exponentially. Therefore to be able to use this model two different time scales were used in the SLP algorithm: A rougher timescale is used to calculate the equality matrices, while a more detailed timescale is used to calculate the states using the Simulink model.

As usual for numerical calculation the exact required position will not be reached. The error tolerance was set to 10^{-3}. For the calculation it seemed to be more important to get a higher accuracy on the position than on the velocity. Since the final state vector consisted of a position and a velocity, a scaling factor $[100; 0.1]$ was used to increase the sensitivity. In this case the error on the position is increased and the error on the velocity is decreased. To compensate any possible differences in the position, an adjustment will be made to the feedback control scheme shown in Fig. 3.

C. Applied Feedback Control

Just using the input profile calculated above would create an open-loop control. The problem of an open-loop control is, that if the system changes its behavior or different initial and final states are used, it might not be usable because the behavior of the system is different. One way to adjust to this problem is to create a feedback control. This feedback control was introduced in sect. II-E.

Since the input profile created in section III-B doesn't exactly reach the desired final state x_f a modification is done on the block diagram from Fig. 3 which is shown in Fig. 6. This block diagram includes a switch logic in the lower right hand corner which forces the real system to reach the exact final position $y_{k,f}$.

The feedback control had to be designed. Since normal control theory approaches from standard control theory books like [4], [5] do not work, a different approach has to be taken. Similar to the optimal search done for the optimal linear control. The control parameters were chosen to minimize the MSE. In this process a PI-control with the

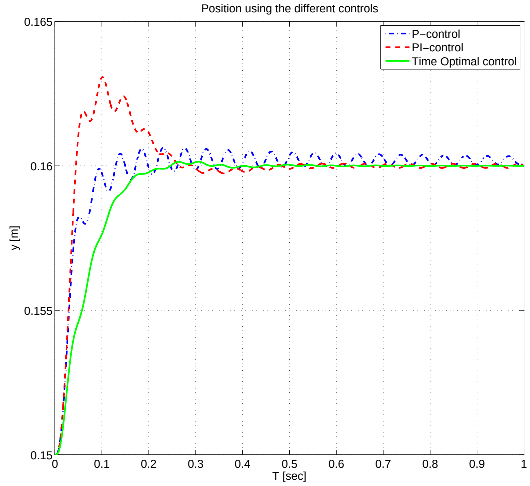

Fig. 7. Position of the load of the hydraulic system

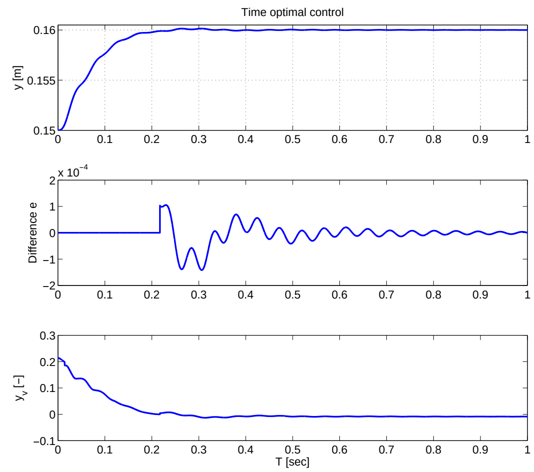

Fig. 8. Position, difference e and input using the time optimal control profile

following parameters have been established.

$$K_P = 40 \quad K_I = 2000 \tag{14}$$

IV. Comparing the two Control Designs

This section compares the results between the different control designs. It will start off by comparing the results for the system earlier defined in sect. II-A. After this comparison the controls will be tested on a system with different parameters.

A. Comparing with the defined system

The controls have been designed in section II-A for the described hydraulic system with a mass load of 500kg. Hydraulic systems are normally used to actuate high masses [2]. Another reason for this relatively high load is, that the system reacts highly oscillatory and the advantage of the optimal control input profile comes more into effect.

The P- and the PI-controls are simulated using a standard feedback control. For the time optimal input profile the modified feedback control block diagram, shown in Fig. 6 is used.

Figure 7 shows the position of the load using the different controls. It can be observed, that the PI-control shows a high overshoot, while the P-control oscillates around the final position. The time optimal input profile reaches the final position and then only shows a very small overshoot of less than 0.1%. Figure 8 shows the position, the difference e between the model and the actual system and the input profile of the hydraulic system for the optimal control. The difference e plot shows, that at approximately $0.21sec$ the time optimal control profile ends and the feedback control compensates the difference between the position reached with the optimal control profile and the desired final position $y_{k,f}$. The peak in the difference e plot shows, that the state reached by the optimal control input profile is 10^{-4} off from the desired final position.

Observing the results shown in Figure 7 the time optimal control combined with the feedback control seems to be the better control for the observed system. Before this conclusion can be truly made, a more detailed analysis has to be done. Since one of the major problems with time optimal control is, that it has specified assumptions for the system in terms of parameters, but also initial and final position needs to be assumed, it has to be analyzed how changes on those assumptions effect the quality of the control. These effects will be studied in section IV-B.

In comparison to an Optimal Control designed using an LQR approach, no choice for the weight matrix is needed. Also the limitation of the control input can only be incorporated with the R-matrix. Therefore the LQR approach is not analysed.

B. Comparing with varying systems

To get a feeling of the robustness of the time optimal control, this section will analyze how the system reacts if the specified assumptions of the system are changed. First the mass will be varied and after that the initial and final position will be altered.

1) Varying the mass: To analyze how the hydraulic system reacts on a change of the mass. The mass will vary between $250\text{kg} - 750\text{kg}$. To be able to compare the controls the performance index ITAE shown in Table I is being used. The results are shown in Fig. 9. These results show, that even for a varying mass the optimal control produces the best results.

2) Varying the initial position: As a further test the initial and final position will be altered. To be able to compare the results the step size between the initial and the final state will be kept constant with the original step size of 0.01. Figure 10 compares the result for an initial position $y_{k,0} = 0.1$ and a final position $y_{k,f} = 0.11$ and Fig. 11 for an initial position $y_{k,0} = 0.2$ and a final position $y_{k,f} = 0.21$. Both figures

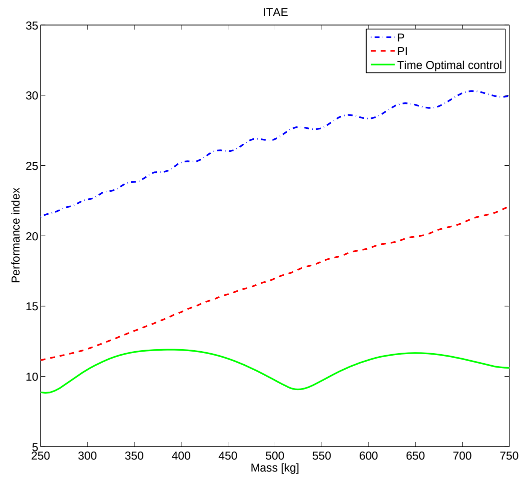

Fig. 9. Performance index for the three control designs for varying mass loads

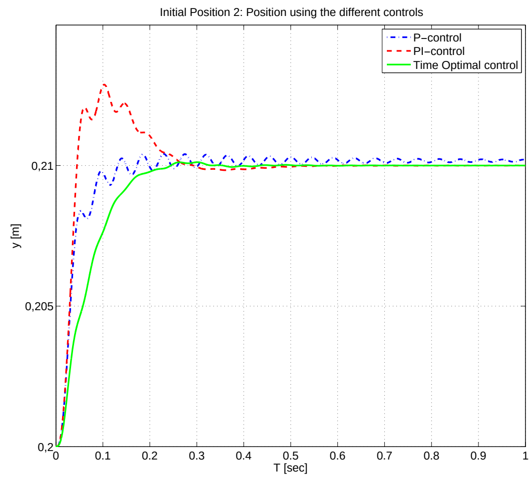

Fig. 11. Position for $y_{k,0} = 0.2$ and $y_{k,f} = 0.21$ using different controls

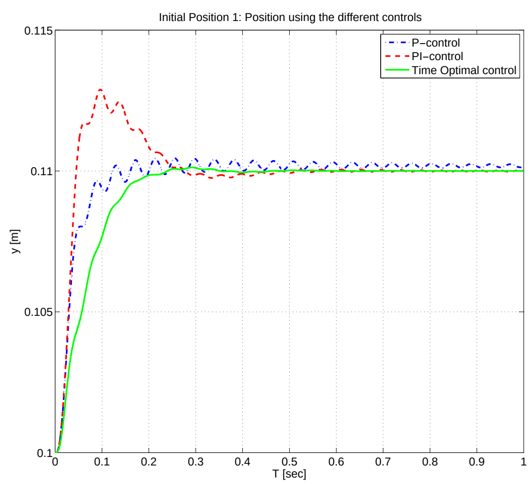

Fig. 10. Position for $y_{k,0} = 0.1$ and $y_{k,f} = 0.11$ using different controls

show that even the initial positions have been changed the time optimal control still shows less overshoot compared to the PI-control and less oscillation compared to the P-control.

V. Conclusion

It can be said, that the proposed approach to use an optimal control design using an SLP approach with a feedback transformation can be used on systems, if the system dynamics and parameters are not exact. Even if the mass of the systems is varied by $\pm 50\%$ the designed optimal control input profile still was usable. One of the goals of this paper was to test the approach on a real system. The existing hydraulic system is currently not able to handle such high masses as 500kg. Testing the approach using a smaller mass such as the existing 53kg will not give the desired results, because the system is not oscillating and therefore the advantage of the optimal control profile will not come that much into effect.

To further extend the SLP algorithm and make it more practicable some recoding can be done to transform the code to a more efficient programming language such as C. Also some tests on real system will give further insight in the advantages but also possible disadvantages for the optimal control.

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