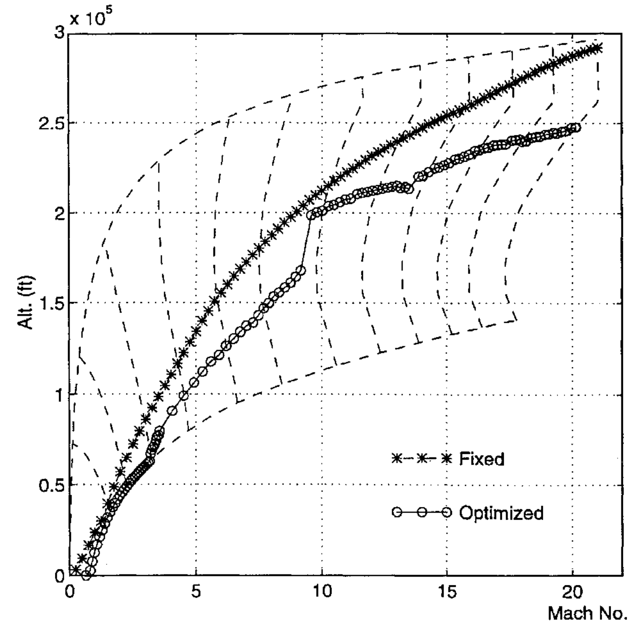

Fig. 2 Fixed and optimized flight paths.

Conclusion

A simple guidance law for operation of dual-fuel SSTO launch vehicles has been developed and used to determine the optimal value of the transition Mach number from dual fuel to single fuel. For the example considered, the optimal transition Mach number was 9.0 along a fixed trajectory. Along an optimal trajectory, the best transition Mach number was 9.6; the optimal trajectory had higher dynamic pressure than the fixed, particularly in dual-fuel mode.

In the future, the guidance method described in this Note easily could be extended to optimize other propulsion system parameters, such as flow rates of individual propellants in multipropellant engines. Because the guidance algorithm is internal to the trajectory optimization routine, its use will save many iterations of a preliminary design computer code relative to treating these parameters as external design variables. The guidance law is highly accurate, robust, and simple to implement, also making it ideal for use in a real-time onboard control system for an SSTO launch vehicles.

Acknowledgment

This work was supported by NASA Ames Research Center Grant NCC 2-5069.

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Effect of Damping on the Structure of the Time-Optimal Control Profile

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I. Introduction

THERE exists a significant body of work that addresses the design of time-optimal controllers. Singh et al.,[1] Ben-Asher et al.[2] Liu and Wie,[3] and Singh and Vadali[4] study the problem of design of time-optimal controllers for flexible slewing structures, which are represented by finite-dimensional linear dynamical models. Liu and Wie[3] and Singh and Vadali[4] also address the problem of desensitizing the time-optimal controller to errors in system parameters. The problem of time-optimal reorientation of spacecrafts has been addressed by Billimoria and Wie,[5] who noted a change in the control structure from a seven-switch to a five-switch profile with an increase in the maneuver. A survey of the problem of time-optimal attitude maneuvers has been provided by Scrivener and Thompson.[6] Pao,[7] via a parametric study of a damped floating oscillator, exemplified the existence of three- and five-switch time-optimal control profiles. The damping ratios corresponding to the transition of the control structure were determined by a parametric study.

This Note proposes a simple technique to determine the transition of the control profile from one structure to another. A damped floating oscillator with the same system parameters as used by Pao[7] is considered to illustrate the proposed technique, where the values of the damping ratio that corresponds to the transition from a three switch to a five switch and, finally, to a three-switch control profile are determined.

II. Existence of Control Profile Transitions

The time optimal control for the system

$$\dot{x} = Ax + bu \tag{1}$$

is given by the equation

$$u = -\text{sgn}(\lambda^T b) \tag{2}$$

where $x \in R^n$ is a vector of system states, $\lambda \in R^n$ is the costates vector, and $u \in R^m$ is the control vector.

It can be seen from Eq. (2) that the control input changes sign when the switching function $\lambda^T b$ passes through zero. With the variation of certain parameters in the control problem, additional switches can be introduced into the control structure either at the initial time or final time or at some time in between the initial and final time. If there exists a control profile transition effected by switches appearing at the initial or final time of the ith control input, the equation that has to be satisfied is

$$(\lambda^T b)_i = 0 \tag{3}$$

at the initial or final time, respectively, since only one switch can enter from either end of the time boundaries. The variation with respect to the system parameters of the initial costates has been assumed to be smooth. If two switches are introduced simultaneously at both ends of the time boundaries, then Eq. (3) has to be simultaneously satisfied at the corresponding time instants. Since the costates are smooth functions of time, if the switches appear in between the initial and final time, then two switches are introduced simultaneously and the transition occurs when the equations

$$(\lambda^T b)_i = 0 \tag{4}$$

Received May 31, 1995; revision received Aug. 8, 1995; accepted for publication March 27, 1996.

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and

$$\left[\frac{\mathrm{d}(\lambda^T b)}{\mathrm{d}t}\right]_i = 0 \tag{5}$$

are satisfied. Equation (4) is the constraint that forces the value of the switching function to be zero at the transition point, and Eq. (5) indicates that the slope of the switching curve at the transition point should also be zero. To illustrate the proposed technique, we consider the benchmark damped floating oscillator example in the next section.

III. Numerical Example

Singh and Vadali[4] have proposed a frequency-domain approach to the design of time-optimal controllers. The proposed technique involves the design of a time-delay filter, the output of which is the time-optimal control profile when it is subject to a step input. The time-delay filter is parametrized in terms of the switch times and the maneuver time, and a parameter optimization problem is formulated that minimizes the maneuver time subject to the constraint that a set of zeros of the time-delay filter cancel all of the poles of the system in addition to the satisfaction of the rigid-body boundary conditions.

Parametrizing the control profile in terms of n switch times and maneuver time T_{n+1}, the design of the time-optimal controller of the benchmark damped floating oscillator (Fig. 1) can be stated as the minimization of

$$J = T_{n+1} \tag{6}$$

subject to the constraints

$$-2T_1 + 2T_2 - \cdots + 2T_{n-1} - 2T_n + T_{n+1} = 0 \tag{7}$$

which ensures the cancellation of the rigid-body poles:

$$1 + 2\sum_{i=1}^{n}(-1)^i e^{-\sigma T_i}\cos(\omega T_i) + e^{-\sigma T_{n+1}}\cos(\omega T_{n+1}) = 0 \tag{8}$$

$$2\sum_{i=1}^{n}(-1)^i e^{-\sigma T_i}\sin(\omega T_i) + e^{-\sigma T_{n+1}}\sin(\omega T_{n+1}) = 0 \tag{9}$$

which corresponds to canceling the complex conjugate poles at $s = \sigma + j\omega$, and

$$\theta(T_{n+1}) = \phi_0\left[\tfrac{1}{2}T_{n+1}^2 - (T_{n+1}-T_1)^2 + (T_{n+1}-T_2)^2 - \cdots 0\right] \tag{10}$$

which satisfies the rigid-body boundary condition. Here, ϕ_0 is the entry of the control influence matrix corresponding to the rigid-body mode of the decoupled equations of motion.[4] The switch times and the maneuver time determined from the solution of the preceding constrained parameter optimization problem are used to determine the initial costates. These costates are then used to determine the switching function, which generates a control profile whose switch times coincide with the ones predicted by the parameter optimization problem for optimality.[4]

Figure 2 illustrates the three different time-optimal control structures that correspond to damping ratios of $\xi = 0.1, 0.2$, and 0.4. As the damping ratio increases, the second and the third switches tend to each other and after some critical value of the damping ratio, two switches are introduced toward the end of the maneuver. Further increase in the damping ratio leads to the coincidence of the second and third switches resulting in a three-switch maneuver. At the instant two switches are introduced and when the two switches collapse, Eqs. (4) and (5) have to be satisfied. Figure 3 illustrates the switching function and the corresponding control profile at the instant of the introduction of the final two switches and the collapse of the second and third switch. The last two switches are introduced when the damping ratio $\xi = 0.1513$, and the second and third switches collapse when $\xi = 0.2247$.

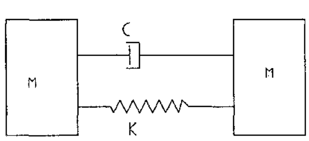

Fig. 1 Damped floating oscillator.

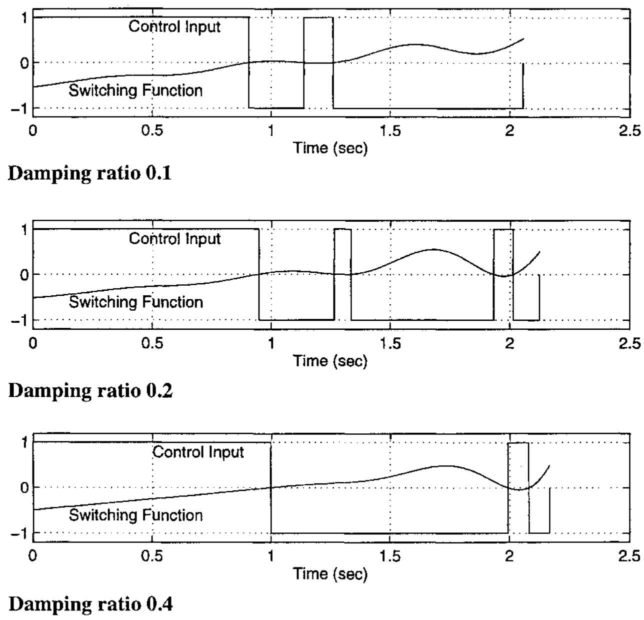

Fig. 2 Control profile structures.

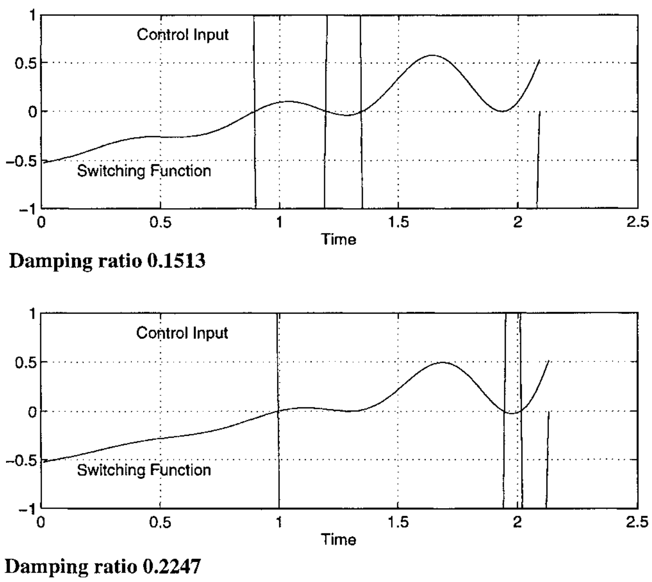

Fig. 3 Transition of control profile structure.

IV. Conclusion

A simple technique for the determination of parameters that correspond to the transition of one control structure to another has been proposed. A damped floating oscillator was used to illustrate the procedure to determine the damping ratio that corresponds to the transition of a three switch to a five switch and a five switch to a three switch control profile. This technique can also be used to determine the maneuver that corresponds to the transition of a five switch to a six switch and finally to a seven switch control profile for a rigid spacecraft.

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Decoupling of the Longitudinal Modes of Advanced Aircraft

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Nomenclature

K_f, t_f, K_e, t_e	= gain and time constant of the flaperon and the elevator actuators
m, I_y, g	= aircraft mass, y-axis moment of inertia, gravitational acceleration
U, W, Q, Θ, Γ $(U_0, W_0, q_0, \vartheta_0, \gamma_0)$	= x, z translation velocities, pitch rate, pitch, and flight-path angle (nominal values)
X, Z, M	= x, z, and y external aerodynamic and propulsion forces and moment
$Z_i, M_i (i = u, \vartheta, w, q, \dot w, \delta_f, \delta_e)$	= dimensional stability derivatives
$\delta_f(t), \delta_e(t), \delta_{cf}(t), \delta_{ce}(t)$	= flaperon and elevator deflections and the respective pilot's commands
$\vartheta(t), \gamma(t), q(t), w(t)$	= pitch, flight-path angle, pitch rate, and vertical velocity increments

I. Introduction

THE coupling between the pilot commands (inputs) and the flight variables to be controlled (outputs) adversely affects the flying qualities. Flight variable mode decoupling with simultaneous satisfactory damping and settling time is one of the central problems in flight control.[1–6] Eigenstructure assignment and input output decoupling appear to be the most suitable design techniques satisfying the requirements for aircraft with many control effectors. According to the eigenstructure assignment technique,[1,2] after selecting an ideal set of closed-loop eigenvalues and corresponding eigenvectors, satisfactory mode decoupling and flying qualities are obtained. In Refs. 4 and 5, using static and dynamic input–output decoupling controllers, independent control between the pitch angle and the downward speed has been performed. Other results in the field can be found in Ref. 3 for the case of an advanced fighter technology integration (AFTI) F-16 aircraft via robust eigenstructure assignment, in Ref. 6 where the model following technique is applied to decoupled flight control, and in Ref. 7 where decoupling and robustness is fulfilled via crossfeed, for the case of rotorcrafts.

The objective of this Note is to control independently the flight-path angle and pitch angle of a multimode aircraft, while preserving satisfactory flying qualities. To meet the benefits of both design techniques (eigenstructure assignment and input–output decoupling), the design technique of input–output decoupling with simultaneous arbitrary pole assignment is proposed. The problem is treated as a generic application facilitating the determination of the class of the stability derivatives for which pitch angle and flight-path angle can be controlled independently via static state feedback. The problem is proven to be solvable for almost all flight conditions, yielding exact decoupling, as well as desirable damping and settling time for the two resulting closed-loop subsystems. Each subsystem is a single-input–single-output all pole system having arbitrary denominator coefficients. These coefficients are the free parameters of the controller matrices. Appropriate tuning of these coefficients leads to adequate short period flying qualities for flying phase categories (A, B, or C). Using the present control scheme, the requirements of pitch pointing and vertical translation maneuvers can easily be met. Finally, all results are illustrated by simulation for an AFTI F-16 aircraft.

II. Model Description

The nonlinear equations describing the longitudinal motion of an aircraft are as follows[8]:

$$X - mg\sin(\Theta) = m(\dot U + QW)$$

$$Z + mg\cos(\Theta) = m(\dot W - QU), \qquad M = I_y\dot Q \tag{1}$$

$$\dot\Theta = Q, \qquad \Theta - \Gamma = \tan^{-1}(W/U)$$

Here we study the longitudinal motion of an advanced aircraft, for straight symmetric flight with wings level. If W_0 is sufficiently small, the equality $\vartheta - \gamma \approx w/U_0$, can be used to derive the following short period approximation:

$$\dot x(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t), \qquad x(0-) = x_0 \tag{2a}$$

with

$$x(t) = [\gamma(t), q(t), \vartheta(t), \delta_f(t), \delta_e(t)]^T$$

$$y(t) = [\gamma(t), \vartheta(t)]^T, \qquad u(t) = [\delta_{cf}(t), \delta_{ce}(t)]^T \tag{2b}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} \tilde Z_w & -\tilde Z_q/U_0 & -\tilde Z_w + (\tilde g/U_0)\sin(\vartheta_0) & -\tilde Z_{\delta f}/U_0 & -\tilde Z_{\delta e}/U_0 \\ -\tilde M_w U_0 & \tilde M_q & \tilde M_w U_0 + \tilde M_\vartheta & \tilde M_{\delta f} & \tilde M_{\delta e} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_f^{-1} & 0 \\ 0 & 0 & 0 & 0 & -t_e^{-1} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_f t_f^{-1} & 0 \\ 0 & K_e t_e^{-1} \end{bmatrix} \tag{2c}$$

where $\tilde M_w = M_w + M_{\dot w}\tilde Z_w$, $\tilde M_q = M_q + (U_0 + \tilde Z_q)M_{\dot w}$, $\tilde M_\vartheta = -\tilde g M_{\dot w}\sin(\vartheta_0)$, $\tilde M_{\delta e} = M_{\delta e} + M_{\dot w}\tilde Z_{\delta e}$, $\tilde M_{\delta f} = M_{\delta f} + M_{\dot w}\tilde Z_{\delta f}$,

$$\tilde Z_w = \frac{Z_w}{1 - Z_{\dot w}}, \qquad \tilde Z_q = \frac{Z_q + U_0 Z_{\dot w}}{1 - Z_{\dot w}}, \qquad \tilde Z_{\delta e} = \frac{Z_{\delta e}}{1 - Z_{\dot w}}$$

$$\tilde Z_{\delta f} = \frac{Z_{\delta f}}{1 - Z_{\dot w}}, \qquad \tilde g = \frac{g}{1 - Z_{\dot w}}$$

Received Sept. 25, 1995; revision received April 17, 1996; accepted for publication April 17, 1996.

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