Jerk Limited Input Shapers

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The focus of this paper is on the design of jerk limited input shapers (time-delay filters). Closed form solutions for the jerk limited time-delay filter for undamped systems is derived followed by the formulation of the problem for damped systems. Since the jerk limited filter involves concatenating an integrator to a time-delay filter, a general filter design technique is proposed where smoothing of the shaped input can be achieved by concatenating transfer functions of first order, harmonic systems, etc. [DOI: 10.1115/1.1653808]

1 Introduction

Prefiltering of command inputs to systems with underdamped modes has been addressed by various researchers [1–4]. Smith's *Posicast Controller* [1] was motivated by a simple wave cancellation concept for the elimination of the oscillatory motion of underdamped systems. This technique required exact knowledge of the damping and natural frequency of the plant to be able to eliminate residual vibrations. Singer and Seering [2] addressed this problem by proposing a technique to design a series of impulses whose amplitudes and application time were determined so as to force the residual energy and the sensitivity of the residual energy with respect to natural frequency or damping to zero. The filtered input was then generated by the convolution of the command input with the impulse sequence. Singh and Vadali [5] proposed a technique to design time-delay filters whose performance was identical to the *Input Shaping* technique proposed by Singer and Seering [2]. Over the past decade numerous papers have been published which deal with the design of discrete time and continuous time prefilters for the robust vibrations control of maneuvering structures. These include the digital shaping filter by Murphy and Watanabe [6], multi-hump input shapers by Singhose et al., minimax filters by Singh [7], user specified time-delay filters by Singh and Vadali [5] besides others. The technique to desensitize the input profile to modeling errors have been used to address a slew of classic optimal control problems such as time-optimal [8–10], fuel-time optimal [11], and minimum power/jerk controllers [12].

The input shaping/time-delay filtering technique include information of specific modes in the design process. If it is necessary to roll off the energy over the high frequency spectrum, additional constraints need to be included in the design process. Jerk limits in the design process can result in control profiles which can be tracked by actuators and which can be used to minimize the excitation of the unmodeled high frequency modes of structures. Muenchhof and Singh [13] present a detailed development of the design technique for the minimum-time jerk limited control profiles for maneuvering underdamped flexible structures. Lim et al. [14], propose a technique for the design of multi-input shapers which permits inclusion of constraints on the jerk. This paper addresses the problem of jerk limited input shapers for prefiltering command inputs to vibratory systems without rigid body modes. The paper by Muenchhof and Singh [13] addressed the problem of design of control profiles for systems with rigid body modes. The paper will start by addressing the design of time-delay filters where the delay time and the gains of the delayed signals are all unknown. This will be followed by the presentation of a general concept to design input shapers by including additional dynamics to the time-delay filter such as harmonic oscillators and first order dynamics to permit smooth ramping up and ramping down of control profiles. The paper will conclude with some remarks.

2 Jerk Limited Input Shapers

2.1 Undamped Systems. This section deals with the design of Jerk Limited Time-Delay filter (Input Shaper) which is schematically represented in Fig. 1. The development which follows is for a single mode system, but can be easily extended for multiple mode systems.

The transfer function of the filter shown in Fig. 1 without the integrator element is

$$G(s)=(1-\exp(-sT_1)+\exp(-s(2T_2-T_1))-\exp(-2sT_2)), \tag{1}$$

The output of the transfer function G(s) subject to a unit step input is shown in Fig. 2 and its time integral is represented as

$$y(t)=J(t-(t-T_1)\mathcal{H}(t-T_1)+(t-(2T_2-T_1))\times\mathcal{H}(t-(2T_2-T_1))-(t-2T_2))\mathcal{H}(t-2T_2))), \tag{2}$$

where J is the permissible jerk and $\mathcal{H}(\ )$ is the Heaviside Step function. $y(t)$ should equal 1 at steady state for a DC gain of unity which results in the constraint equation

$$y(2T_2)=J(2T_2-(2T_2-T_1)+(2T_2-(2T_2-T_1)))=1, \tag{3}$$

or

$$T_1=\frac{1}{2J} \tag{4}$$

which implies that the first switch T_1 is only a function of the permitted jerk. To cancel the undamped poles of the system, we require a pair of zeros of the time-delay filter to cancel the poles of the system. This results in the constraint equations

$$1-\cos(\omega T_1)+\cos(\omega(2T_2-T_1))-\cos(2\omega T_2)=0 \tag{5}$$

and

$$-\sin(\omega T_1)+\sin(\omega(2T_2-T_1))-\sin(2\omega T_2)=0 \tag{6}$$

These two constraint equations are satisfied if

$$\sin(\omega T_2)=\sin(\omega(T_2-T_1)). \tag{7}$$

Substituting Eq. 4 into Eq. 7, and simplyfing we have

$$\tan(\omega T_2)=-\cot\left(\frac{\omega}{4J}\right) \tag{8}$$

which results in the closed form solutions

$$T_2=\frac{(2n+1)\pi}{2\omega}+\frac{1}{4J}. \tag{9}$$

For specific values of ω and J, T_1 can equal T_2, which corresponds to the first and the second switch collapsing. From Eqs. 4 and 7, this corresponds to

$$\sin(\omega T_2)=0, \quad \Rightarrow\cos\left(\frac{\omega}{4J}\right)=0 \tag{10}$$

or

$$\frac{\omega}{4J}=(2m+1)\frac{\pi}{2}, \quad \mathrm{m}=1,2,3\ldots \tag{11}$$

So, for a given J or ω, we can solve for ω or J respectively for which $T_1 T_2$ are equal, which corresponds to a simple ramp input to the system.

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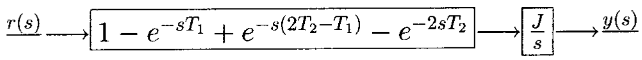

Fig. 1 Single Time-Delay Controlled System

Figures 3 and 4 illustrate the variation of the switch times and the final time of the time-delay filter as a function of varying frequency and Jerk respectively. It is clear from Fig. 4, that the first and second switch coincide which corresponds to the solid line intersecting the dashed line. Figure 4 is generated for $J=3$, for which we have from Eq. 11, $\omega=6\pi$, 18π, 30π, for which the switches collapse, which corroborates the results in Fig. 4.

2.2 Damped Systems. The jerk limited time delay filter for damped systems cannot be solved in closed form. The problem can be solved numerically by an optimization problem.

The jerk limited time-delay filter is parameterized as

Fig. 2 Parameterized Control Profile

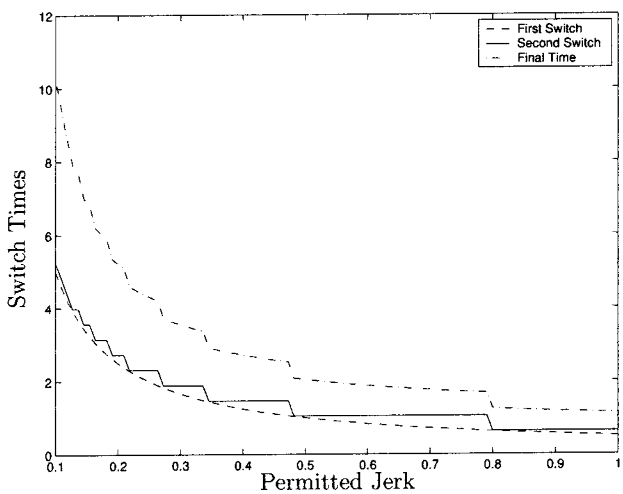

Fig. 3 Switch Time Variation vs Jerk for ω=15

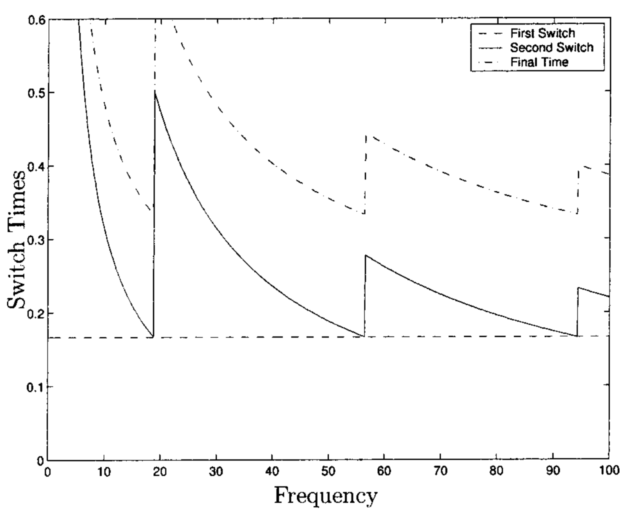

Fig. 4 Switch Time Variation vs Frequency

$$G(s)=\frac{J}{s}(1-\exp(-sT_1)+\exp(-sT_2)-\exp(-sT_3)). \tag{12}$$

To satisfy the requirement that the final value of the jerk limited time-delay filter be unity when it is driven by an unit step input results in the constraint equation

$$y(T_3)=J(T_3+T_1-T_2)=1. \tag{13}$$

To cancel the damped poles of the system at $s=\sigma\pm j\omega$, we require a pair of zeros of the time-delay filter to cancel the damped poles of the system. This results in the constraint equations

$$1-e^{-\sigma T_1}\cos(\omega T_1)+e^{-\sigma T_2}\cos(\omega T_2)-e^{-\sigma T_3}\cos(\omega T_3)=0 \tag{14}$$

and

$$-e^{-\sigma T_1}\sin(\omega T_1)+e^{-\sigma T_2}\sin(\omega T_2)-e^{-\sigma T_3}\sin(\omega T_3)=0. \tag{15}$$

The optimization problem can be stated as minimization of T_3 subject to the three equality constraints given by Eqs. 13, 14, and 15.

3 Robust Jerk Limited Time-Delay Filter

Most systems have errors in estimated damping and natural frequencies which can result in significant residual errors when a rest-to-rest maneuver is performed. It is therefore imperative to design filters which can handle uncertainties in estimated model parameters. There are multiple approaches to achieve robustness. The simplest includes reducing the sensitivity of the residual energy of the modes, at the nominal values of estimated system parameters. If bounds and distributions of the uncertain parameters are available to the designer, the minimax approach proposed by Singh [4] can be used to arrive at filters which minimize the maximum magnitude of the residual energy in the domain of interest. In this work, robustness is achieved by placing multiple zeros of the time-delay filter at the location of the uncertain poles of the plant.

The added requirement of robustness results in a filter with increased number of parameters to be determined. The approach for the design of robust jerk limited time-delay filters is developed for damped systems with the knowledge that the undamped systems are a sub-set of the damped system. The robust jerk limited time-delay filter is parameterized as

$$G(s)=\frac{J}{s}(1-\exp(-sT_1)+\exp(-sT_2)-\exp(-sT_3)+\exp(-sT_4)-\exp(-sT_5)). \tag{16}$$

To satisfy the requirement that the final value of the jerk limited time-delay filter be unity when it is driven by an unit step input results in the constraint equation

$$y(T_5)=J(T_5-T_4+T_3-T_2+T_1)=1. \tag{17}$$

To cancel the damped poles of the system at $s=\sigma\pm j\omega$, we require a pair of zeros of the time-delay filter to cancel the damped poles of the system. This results in the constraint equations

$$1-e^{-\sigma T_1}\cos(\omega T_1)+e^{-\sigma T_2}\cos(\omega T_2)-e^{-\sigma T_3}\cos(\omega T_3)+e^{-\sigma T_4}\cos(\omega T_4)-e^{-\sigma T_5}\cos(\omega T_5)=0 \tag{18}$$

and

$$-e^{-\sigma T_1}\sin(\omega T_1)+e^{-\sigma T_2}\sin(\omega T_2)-e^{-\sigma T_3}\sin(\omega T_3)+e^{-\sigma T_4}\sin(\omega T_4)-e^{-\sigma T_5}\sin(\omega T_5)=0. \tag{19}$$

The robustness is achieved by placing a second pair of zeros of the time-delay filter at the estimated location of the oscillatory poles of the system, which results in the equations

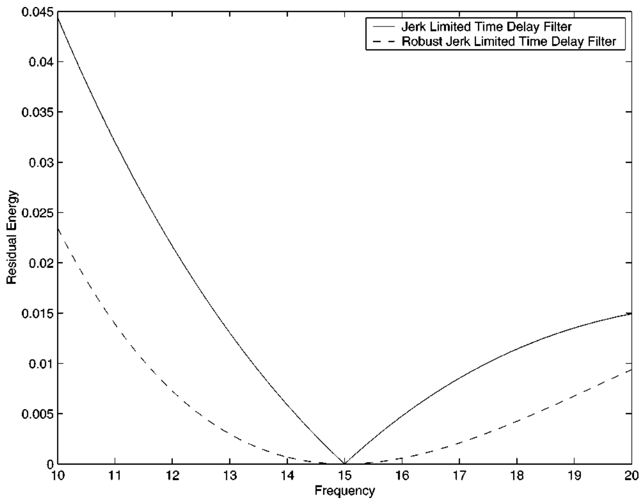

Fig. 5 Sensitivity Curve

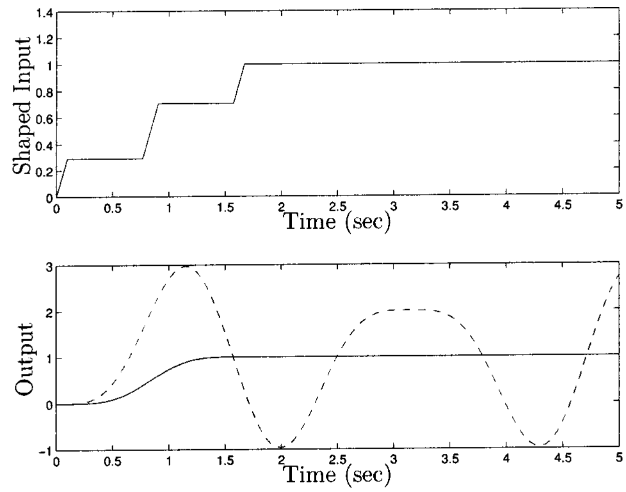

Fig. 6 Shaped Input and Comparison of system response

$$-T_1e^{-\sigma T_1}\sin(\omega T_1)+T_2e^{-\sigma T_2}\sin(\omega T_2)-T_3e^{-\sigma T_3}\sin(\omega T_3)+T_4e^{-\sigma T_4}\sin(\omega T_4)-T_5e^{-\sigma T_5}\sin(\omega T_5)=0 \tag{20}$$

and

$$-T_1e^{-\sigma T_1}\cos(\omega T_1)+T_2e^{-\sigma T_2}\cos(\omega T_2)-T_3e^{-\sigma T_3}\cos(\omega T_3)+T_4e^{-\sigma T_4}\cos(\omega T_4)-T_5e^{-\sigma T_5}\cos(\omega T_5)=0. \tag{21}$$

The optimization problem can now be stated as the minimization of T_5 subject to the constraint given by Eqs. 17–21.

To illustrate the reduced sensitivity of the residual energy to variations in the frequency, the response of the system was studied for various values of model frequencies with a filter designed for a frequency of 15 rad/s and a permitted jerk of 4. Figure 5 illustrates the improved performance of the robust jerk limited time-delay filter.

4 Jerk Limited Time-Delay Filters for Multimode Systems

The proposed approach can be used for the control of systems with multiple under-damped modes. A generic formulation is developed below. The number of parameters to be optimized for can be reduced for undamped systems by exploiting the symmetric characteristics of the time-delay filter. The transfer function of the time-delay filter is

$$G(s)=\frac{J}{s}\sum_{i=0}^{N}(-1)^i\exp(-sT_i) \tag{22}$$

where $T_0=0$ and N is an odd number. The unknowns (T_i) have to satisfy the constraint equation

$$\sum_{i=1}^{N}(-1)^{i+1}T_i=\frac{1}{J} \tag{23}$$

which satisfies the requirement that the final value of the output of the filter when it is subject to an unit step input is unity. To cancel the undamped or under-damped poles at

$$s_k=\sigma_k\pm j\omega_k \quad \text{for } k=1,2,3,\ldots \tag{24}$$

the following constraints have to be satisfied

$$\sum_{i=1}^{N}(-1)^i\exp(-\sigma_kT_i)\cos(\omega_kT_i)=0 \quad \text{for } k=1,2,3,\ldots \tag{25}$$

and

$$\sum_{i=1}^{N}(-1)^i\exp(-\sigma_kT_i)\sin(\omega_kT_i)=0 \quad \text{for } k=1,2,3,\ldots \tag{26}$$

The optimal solution is one which satisfies all the constraints and minimizes T_N.

To desensitize the filter to errors in estimated damping or frequency, the following constraint equations are added to the optimization problem

$$\sum_{i=1}^{N}(-1)^iT_i\exp(-\sigma_kT_i)\sin(\omega_kT_i)=0 \quad \text{for } k=1,2,3,\ldots \tag{27}$$

and

$$\sum_{i=1}^{N}(-1)^iT_i\exp(-\sigma_kT_i)\cos(\omega_kT_i)=0 \quad \text{for } k=1,2,3,\ldots \tag{28}$$

It can be seen that desensitizing the filter with respect to damping simultaneously desensitizes the filter to the frequency as well.

The design of jerk limited time-delay filters for user specified time-delays follows the process proposed by Singh and Vadali [5]. It is clear that additional number of delays are required since the delay times are no longer variables in the optimization process.

To illustrate the design of multi-mode jerk limited input shapers, consider the system

$$\frac{y(s)}{u(s)}=\frac{225}{s^4+34s^2+225} \tag{29}$$

which is characterized by two modes with frequencies 3 and 5. For a jerk constraint of 3, the jerk limited input shaper is designed. The dashed line and the solid line in Fig. 6 illustrates the response of the system to a step input and the shaped input respectively. It is clear that the residual vibration of the two modes is eliminated after shaping the input.

5 Filtered Input Shapers

The technique presented in this work where an integrator is concatenated to a time-delay filter to satisfy the constraint of jerk limited filter design can be extended by cascading other transfer functions such as that of first order systems, harmonic systems etc.

5.1 First-Order Filtered Input Shaper. Instead of using an integrator in conjunction with a time-delay filter to account for the limit on the permitted jerk, one can concatenate a first order

$$r(s) \longrightarrow \boxed{-\frac{e^{aT}}{1-e^{aT}} + \frac{1}{1-e^{aT}}e^{-sT}} \longrightarrow \boxed{\frac{a}{s+a}} \longrightarrow \boxed{A_0 + A_1 e^{-sT_1}} \longrightarrow u(s)$$

Fig. 7 First-Order Filtered Time-Delay Filter

$$r(s) \longrightarrow \boxed{A_0 + A_1 e^{-sT_1} + A_2 e^{-sT_2} + A_3 e^{-sT_3}} \longrightarrow \boxed{\frac{\omega^2}{s^2+\omega^2}} \longrightarrow u(s)$$

Fig. 8 Sinusoid Filtered Time-Delay Filter

$$r(s) \longrightarrow \boxed{0.5 + 0.5e^{-s\pi/\omega}} \longrightarrow \boxed{\frac{\omega^2}{s^2+\omega^2}} \longrightarrow \boxed{A_0 + A_1 e^{-sT_1}} \longrightarrow u(s)$$

Fig. 9 Sinusoid Filtered Time-Delay Filter

filter to a time-delay filter to generate a smooth input which can then be used to drive a time-delay filter designed to cancel the underdamped poles of the system of interest. Figure 7 illustrates the proposed filter structure where T is a user selected time-delay which in the case of a discrete time implementation, can be an integral multiple of the sampling interval.

5.2 Sinusoid Filtered Input Shaper. Filtering with a transfer function of a scaled sinusoid results in an input which emulates a step input but with zero initial and final slopes. The scaling of the sinusoid transfer function is to satisfy the requirement that the DC gain of the transfer function is unity. The sinusoid filtered time-delay filter is illustrated in Fig. 8 which can be rewritten as shown in Fig. 9.

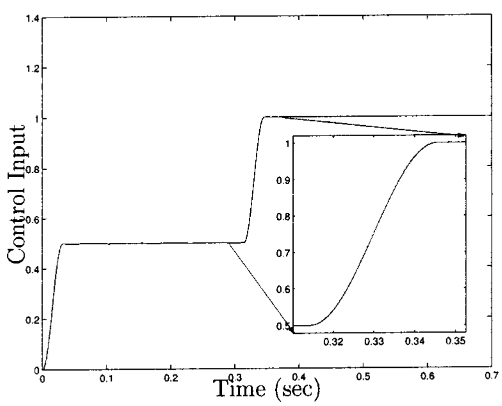

Fig. 10 Sinusoid Filtered, Input Shaped Control Input

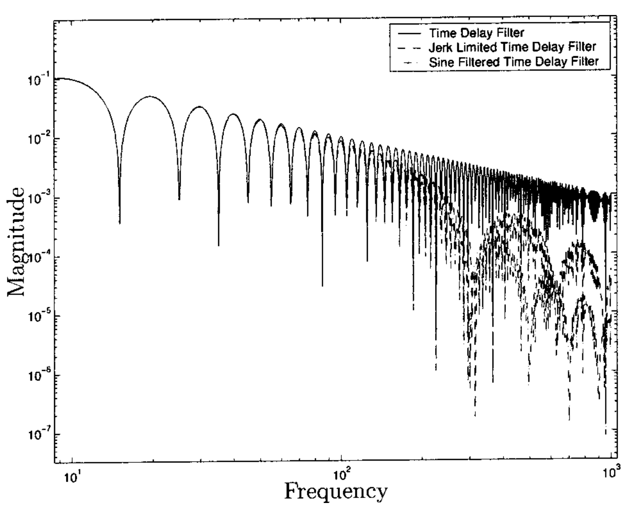

Fig. 11 Frequency Response

$$r(s) \longrightarrow \boxed{0.5 + 0.5e^{-s\pi/\omega}} \longrightarrow \boxed{\frac{\omega^2}{s^2+\omega^2}} \longrightarrow p(s)$$

Fig. 12 Time-Delay Filter

Here the first time-delay filter cancels the oscillatory response of the scaled harmonic oscillator. This truncated harmonic response is then input to the second time-delay filter which is designed to cancel the oscillatory mode of the system. Figure 10 illustrates the control profile. The benefit of this approach can be gauged from the frequency response plots of the sinusoid filtered time-delay filter. Figure 11 illustrates the frequency response plots of the time-delay filter, jerk limited time-delay filter and a sinusoid filtered time-delay filter. The sinusoid filtered time delay filter has been designed such that the maximum jerk of the control profile is equal to the maximum permitted jerk. It can easily be seen that the magnitude plot of the sinusoid filtered time-delay filters rolls off much more rapidly compared to the time-delay filter and the jerk limited time-delay filter. Thus, this input will not significantly excite the unmodeled dynamics.

5.3 Jerk Limits. Consider a part of the sinusoid filtered time-delay filter illustrated in Fig. 12.

The output p of the time-delay filter subject to a unit step input is

$$p(t) = \sin^2(\omega/2t) + \sin^2(\omega/2(t-\pi/\omega))\mathrm{H}\left(t-\frac{\pi}{\omega}\right) \tag{30}$$

and the rate of change of p which is the jerk is

$$\dot{p}(t) = \frac{\omega}{2}\sin(\omega t) + \frac{\omega}{2}\sin(\omega(t-\pi/\omega))\mathrm{H}\left(t-\frac{\pi}{\omega}\right) \tag{31}$$

which implies that the maximum magnitude of the jerk is $\omega/2$ and occurs at time $t=\pi/2\omega$. This is the upper bound for the jerk. It can be seen that the jerk is zero at the start and the end of the maneuver which results in a very practical control profile. When the signal p is passed through the second time-delay filter, based on the damping present in the oscillatory pole to be cancelled, the jerk can lie in the limit

$$\frac{\omega}{4} \leqslant \text{Maximum Jerk} \leqslant \frac{\omega}{2}. \tag{32}$$

If the pole to be cancelled is undamped the maximum jerk is $\omega/4$ since A_0 and A_1 are equal to 0.5. When the poles to be cancelled contain damping, A_0 is greater than 0.5 and A_1 is less than 0.5, resulting in the maximum jerk lying in the range specified by Eq. 32.

This constraint is valid when the time-delay filter is designed to cancel the unwanted under-damped poles. However, if the under-damped pole has to be controlled using a robust time-delay filter, the limits on the jerk changes, since the robust time-delay filter uses smaller gains.

6 Conclusions

A simple technique to design filtered Input Shapers is proposed in this paper. The paper first addresses the problem of design of jerk limited time-delay filters which results in a ramping of the control input. This motivates the design of filtered Input shapers by concatenating transfer functions of a scaled harmonic oscillator in addition to others, to result in smooth control profiles. The roll off of the frequency response plots for the filtered Input Shapers is used to illustrate their benefits.

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Extended Influence Coefficient Method for Rotor Active Balancing During Acceleration

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Imbalance-induced vibration of rotating machineries is an important factor limiting the performance and fatigue life of a rotor system. Particularly, the severe resonant vibration of a rotor when it passes through its critical speeds could damage the rotor system. To avoid this peak vibration, this paper presents an active balancing method to offset the imbalance of the rotor system during acceleration by using an electromagnetic balancer. In this method, "instantaneous" influence coefficients at different speeds are obtained and stored in a look-up table. Then, a gain scheduling strategy is adopted to suppress the imbalance-induced vibration during acceleration based on the "instantaneous" influence coefficient table. A comprehensive testbed is built to validate this scheme, and the validation results are presented.
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1 Introduction

Rotating machinery is commonly used in industries. Imbalance-induced vibration is an important factor limiting the performance and fatigue life of the rotating system. Many balancing procedures have been developed to suppress this imbalance-induced vibration. Among all these methods, off-line balancing methods [8] are widely adopted in practice. However, off-line balancing methods cannot be used if the distribution of imbalance and/or the effective imbalance change during operation. For example, in high speed machining, tool changes frequently happen during the operation of the machine. Different toolholder has different imbalance distribution. In order to overcome this limitation of off-line balancing, some researchers [3,4,6,7,10] tried to actively balance the rotating systems during operation using mass redistribution devices. All these methods require that the rotating speed of the rotor is constant. In some other cases, the balancing needs to be completed during speed-varying transient time in order to save time and get better performance. For example, in high-speed machining, the machining tool will be engaged in cutting as soon as the spindle reaches steady state speed. If an active balancing scheme is used on this machine, the balancing has to be done during the acceleration period to avoid increasing the cutting cycle time. Furthermore, the maximum vibration of a rotor usually occurs when it passes through its critical speeds. To avoid this hostile peak vibration, balancing during acceleration is needed.

Some technical challenges are associated with active balancing during acceleration. First, in the constant rotating speed case the imbalance-induced vibration only contains a single frequency (the rotating speed). Hence, a simple rotor model [2] can be used to develop the active balancing algorithm. However, in the acceleration case the overall dynamics of the rotor are excited. It can be shown that under certain conditions, the speed-varying transient response of a rotor system is quite different from the constant speed response [9,11]. A more comprehensive rotor model needs to be developed to depict the rotor system. Second, it is well known that the responses of rotor systems to imbalance are different at different rotating speeds. To successfully balance the rotor during speed transient period, we need a quick response actuator to catch up with the rotating speed change.

In this paper, an active balancing scheme that can balance the rotor-bearing system during acceleration period is presented. The actuator used in this research is a new type of mass redistribution device [1]. The mass redistribution of this balancer can be finished in fractions of a second. To describe the overall dynamics of the rotor system during acceleration period, influence coefficients at different speeds are obtained and stored in a look-up table. Then,

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