Journal of Dynamic Systems, Measurement and Control

Technical Briefs

Robust Time-Delay Control

T. Singh[1] and S. R. Vadali[1]

A method is presented to minimize residual vibration of structures or lightly damped servomechanisms. The method, referred to as the proportional plus multiple delay (PPMD) control, involves the use of multiple time delays in conjunction with a proportional part to cancel the dynamics of the system in a robust fashion. An interesting characteristic of the controller involves addition of a basic single time-delay control unit in cascade to the existing controller, for every additional requirement of robustness. It is shown that the proposed time-delay controller produces results that are exactly the same as those obtained by the shaped input technique. In addition, it is simpler to arrive at the relative amplitudes of the time-delayed signals for any number of delays even in a multi-input setting.

1 Introduction

With the current interest in the space station, retargeting space structures, and space based robots, there have been a number of studies in the control of vibrations induced by rapid reorientation of the structure itself or its subsystems. Considerable research effort has gone into design of closed and open-loop control strategies. A fairly comprehensive treatment of this family of problems has been presented by Junkins and Turner (1986).

A very interesting technique was proposed by Tallman and Smith (1958) which involved splitting the input excitation into several segments such that the sum of all transient terms is equal to zero after the last excitation. This technique was referred to as the *posicast technique*. This work acknowledged the lack of robustness to errors in estimated damping and frequency of the controlled system. This controller can also be represented as a proportional and time-delayed block. Singer and Seering (1990) proposed a technique for decreasing the sensitivity of the posicast controller to modeling errors. The design of the controller involved studying the response of the system to a sequence of impulses. The amplitudes and times of application of the impulses were determined by solving a set of equations representing the response of the system to the impulse sequence and for robustness, derivatives of these equations with respect to frequency or damping. Singhose et al. (1990) used the phasor diagram to arrive at the same solution. The input shaping technique discussed above is designed to suppress residual vibration of one mode. To eliminate residual vibration due to multiple modes, the impulse sequence for each mode is convolved to produce a new impulse sequence. Wie and Liu (1990) used the shaped input technique to modify the flexible-body time optimal control profile to produce a robust control scheme which was applied to the control of a two-mass spring problem. Hyde and Seering (1991) developed a numerical technique for the design of a shaped input controller for multimode systems, which is simple to implement and produces a smaller system response delay as compared to a cascade of shaped input controllers designed for individual modes. This technique involved the direct solution of a group of simultaneous nonlinear impulse constraint equations. Singh and Heppler arrived at a method for suppressing vibrations for systems with multiple modes, provided certain constraints related to the antisymmetric periodic property of the signal are satisfied. Two impulses for the nonrobust case and three or more impulses for the more robust case are required for this controller.

In this paper, we first design a single time-delay controller and show its equivalence to the two impulse shaped input controller. We then design two and three time-delay controllers and show that they are equal to the three and four impulse shaped input controllers, respectively. A simple generalization is provided to arrive at the relative amplitudes of the time-delay controllers for arbitrary numbers of time delays. The effect of parameter error on the response is also presented.

2 Time-Delay Control

2.1 Proportional Plus Delay (PPD) Control. Open-loop control of a simple harmonic system using time-delay control can be represented as

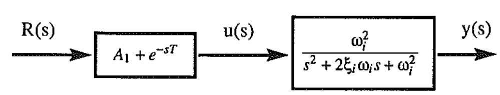

Fig. 1 Time-delay controlled system

The zeros of the controller for the system represented in Fig. 1 are given by the equation

$$A_1 + \exp(-sT) = 0 \tag{1}$$

where we have normalized the relative amplitudes of the proportional and time-delayed signal. A_1 is the amplitude of the proportional signal and T is the delay time of the time-delayed signal.

Representing the Laplace variable s as

$$s = \sigma + j\omega \tag{2}$$

[1]Post Doctoral Fellow and Associate Professor, respectively, Aerospace Engineering, Texas A&M University, College Station, Texas 77843-3141.

Contributed by the Dynamic Systems and Control Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Dynamic Systems and Control Division January 23, 1992; revised manuscript received July 1992. Associate Technical Editor: A. Ray.

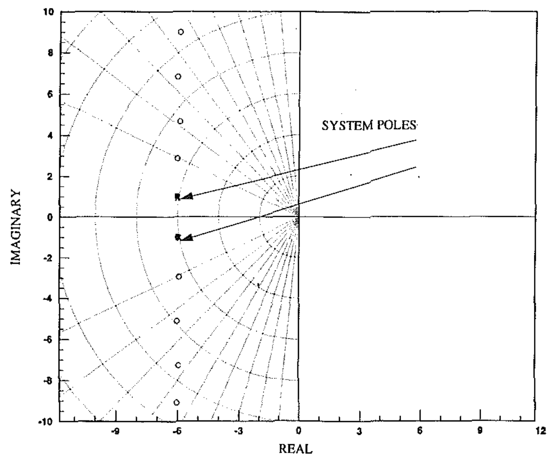

Fig. 2 Pole-zero locations of the controlled system in the *s*-plane

and substituting Eq. (2) into Eq. (1) and equating the real and imaginary parts to zeros, we have

$$A_1 + \exp(-\sigma T)\cos(\omega T) = 0 \tag{3}$$

and

$$\exp(-\sigma T)\sin(\omega T) = 0 \tag{4}$$

From Eq. (4) we have

$$\omega = (2n+1)\frac{\pi}{T},\quad 2n\frac{\pi}{T}\, n = -\infty, \ldots 0, \ldots \infty \tag{5}$$

Substituting Eq. (5) into Eq. (3), the zeros of the controller are

$$s = \begin{cases} \dfrac{-\ln(A_1) + (2n+1)\pi j}{T} & \text{when } \omega = (2n+1)\dfrac{\pi}{T} \\ \dfrac{-\ln(-A_1) + 2n\pi j}{T} & \text{when } \omega = 2n\dfrac{\pi}{T} \end{cases}$$

$$n = -\infty, \ldots, 0, \ldots \infty \tag{6}$$

where

$$A_1 = \pm\exp(-\sigma T) \tag{7}$$

is derived from Eq. (3). In this work we assume A_1 is positive and use the zeros corresponding to that assumption.

To cancel the system poles at $s = -\xi_i\omega_i \pm j\omega_i\sqrt{1-\xi_i^2}$, we have from Eq. (5) (setting $n=0$)

$$\omega = \omega_i\sqrt{1-\xi_i^2} = \frac{\pi}{T} \tag{8}$$

$$\Rightarrow T = \frac{\pi}{\omega_i\sqrt{1-\xi_i^2}} \tag{9}$$

and

$$\sigma = -\xi_i\omega_i. \tag{10}$$

Substituting Eq. (9) into Eq. (3), we have

$$-\ln(A_1) = -\xi_i\omega_i\frac{\pi}{\omega_i\sqrt{1-\xi_i^2}} \tag{11}$$

which leads to

$$A_1 = \exp\left(\frac{\xi_i\pi}{\sqrt{1-\xi_i^2}}\right) \tag{12}$$

which corresponds exactly to the solution of the *shaped-input* technique [5]. The controller can also be written as

$$u(s) = (s^2 + 2\xi_i\omega_i s + \omega_i^2)(s^2 + 2\xi_i\omega_i s + 9\omega_i^2 - 8\xi_i^2\omega_i^2)$$
$$\cdots(s^2 + 2\xi_i\omega_i s + n^2\omega_i^2 - (n^2-1)\xi_i^2\omega_i^2)R(s)$$
$$n = 1,3,5,\ldots. \tag{13}$$

The pole zero location of the controlled system is given in Fig. 2. Thus the single time-delay controller can also be used to cancel poles of the system that are odd multiples of the two primary poles.

The final value of the single time-delayed controlled system to a unit step input, is given by

$$\lim_{s\to 0}\frac{1}{s}\, s\left(\frac{(A_1 + \exp(-sT))\omega_i^2}{s^2 + 2\xi_i\omega_i s + \omega_i^2}\right) \tag{14}$$

which equals

$$A_1 + 1 \tag{15}$$

To force the final value of the input after passing through the controller to be the same as that entering the controller, we normalize the amplitudes of the direct and time-delayed signal, so that the time-delayed controller is

$$\frac{A_1 + \exp(-sT)}{A_1 + 1} \tag{16}$$

2.2 Proportional Plus Multiple Delay (PPMD) Control. The single time-delay control, by virtue of canceling the poles corresponding to the oscillatory behavior of the system, provides us with a technique to produce nonoscillatory response. The cancellation of the dynamics of the system is contingent on an exact estimation of the poles of the system. To ameliorate the robustness of the time-delay control to errors in estimated pole, the double time-delay control is proposed. The controlled system is represented as

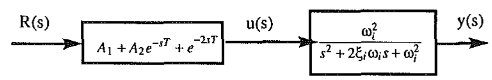

Fig. 3 Time-delay controlled system

With the additional constraint that the derivative of the transfer function of the controller with respect to frequency should go to zero, the transfer function of the controller can be shown to be

$$\frac{u(s)}{R(s)} = \left(\exp\left(\frac{\pi\xi_i}{\sqrt{1-\xi_i^2}}\right) + \exp(-sT)\right)^2 \tag{17}$$

which is two, single time-delay controllers in series.

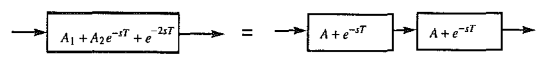

Fig. 4 Time-delay controlled system

where

$$A = \exp\left(\frac{\pi\xi_i}{\sqrt{1-xi_i^2}}\right) \tag{18}$$

and

$$T = \frac{(2n+1)\pi}{\omega_i\sqrt{1-\xi_i^2}} \tag{19}$$

Likewise the three time-delay controller

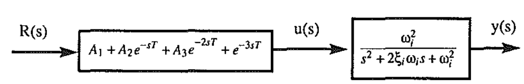

Fig. 5 Time-delay controlled system

has a transfer function

$$\frac{u(s)}{R(s)} = \left(\exp\left(\frac{\pi\xi_i}{\sqrt{1-\xi_i^2}}\right) + \exp(-sT)\right)^3 \tag{20}$$

which is equivalent to three, single time-delay controllers in series.

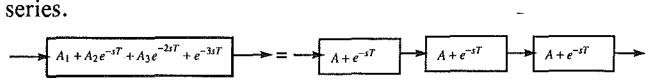

Fig. 6 Time-delay controlled system

This leads us to the observation that the addition of an impulse to a shaped-input controller is equivalent to adding a single time-delayed block. With this observation, we can arrive at the relative amplitudes of the impulse easily for any impulse train sequence.

3 Robustness in the Sense of Bode

The two time-delay controlled system can be represented as

$$\frac{y(s)}{R(s)}=\frac{(s^2+2(\sigma+\epsilon)s+(\omega+\epsilon_1)^2)^2\cdots(s^2+2(\sigma+\epsilon)s+n^2(\omega+\epsilon_1)^2-(n^2-1)(\sigma+\epsilon)^2)^2\omega^2}{s^2+2\sigma s+\omega^2} \qquad n=1,3,\ldots,\infty \qquad (21)$$

where ϵ and ϵ_1 are the errors in estimated damping constant and natural frequency, respectively. The zeros of the controller are given by

$$C=(s^2+2(\sigma+\epsilon)s+(\omega+\epsilon_1)^2)^2\cdots(s^2+2(\sigma+\epsilon)s+n^2(\omega+\epsilon_1)^2-(n^2-1)(\sigma+\epsilon)^2)^2=0 \qquad (22)$$

It can be seen that

$$\frac{dC}{d\epsilon}=0 \qquad (23)$$

and

$$\frac{dC}{d\epsilon_1}=0 \qquad (24)$$

at the zeros of the controller. The implication of Eqs. (23) and (24) can be exemplified using Fig. 7 for a system with $\omega = 1$ and $\sigma = 0.02$. From this figure which illustrates the magnitude plot of the Bode diagram for $\epsilon = \epsilon_1 = 0$, it is seen that the system pole does not produce the classic bell curve because the 1-time-delay controller cancels the system dynamics exactly. The two-time-delay controlled system has a plot that tends to zero at the damped natural frequency because of multiple zero characteristic of the controller. Further the three-time-delay controlled system has a plot that has a slope that tends to zero in addition to a value that further tends to zero at the frequency corresponding to the damped natural frequency of the system. In other words, the curve represent a reduction in the energy content in the frequency spectrum with increase in the number of delays in the controller.

The robustness in the sense of Bode can be illustrated by the magnitude plot of the Bode diagram (Fig. 8) for controllers with 10 percent error in estimated frequency. It is seen that for the one-time-delay controlled system, there exists a spike in the curve at the frequency corresponding to the natural frequency of the system and that there is a large reduction in the magnitude for the two-time-delay controlled system. The three and four-time-delay controlled system has curves that are essentially the same as when the frequency was exactly estimated. The above observation helps us conclude that the addition of a time-delayed block which adds an infinite set of zeros coincident with the previous zeros makes the controller robust as higher derivatives of C with respect to either ϵ or ϵ_1 can be forced to zero.

4 Conclusions

A simple and elegant control strategy has been proposed in this paper to attenuate residual vibrations in mechanical systems. It has been shown that the new PPMD controller produces results that are exactly the same as the shaped input technique. The accent of this paper is on the ease of derivation of the relative amplitudes of the proportional and time-delayed signals. In addition, the interpretation of robustness of the time-delayed/shaped-input controller in the Bode sense has been provided.

The proposed control strategy can be used to control multiple modes by building a system of time-delay controllers in cascade, with each time-delayed controller catering to one mode. The normalized controllers show a decrease in the maximum amplitudes of the proportional and time delayed components with increase in the number of delays. This fact can be used to utilize a smaller actuator with the additional benefit of robustness.

Acknowledgments

This material is based in part upon work supported by the Texas Advanced Technology Program under grant No. 1991/264.

References

Hyde, J. M,. and Seering, W. P., 1991, "Multiple Mode Vibration Suppression in Controlled Flexible Systems," MIT Space Engineering Research Center Report, SERC #3-91.

Junkins, J. L., and Turner, J. D., *Optimal Spacecraft Rotational Maneuvers*, Elsevier, Amsterdam, 1986.

Singer, Neil, C., Seering, C., and Warren, P., 1990, "Preshaping Command

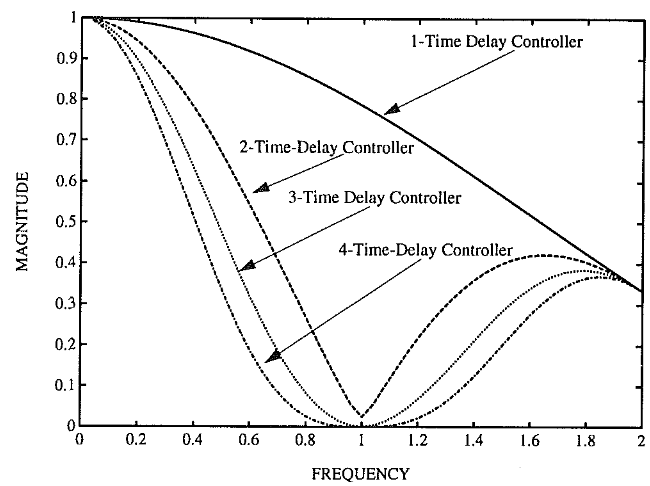

Fig. 7 Magnitude plots of bode diagrams for the time-delay controllers with exact estimated frequency

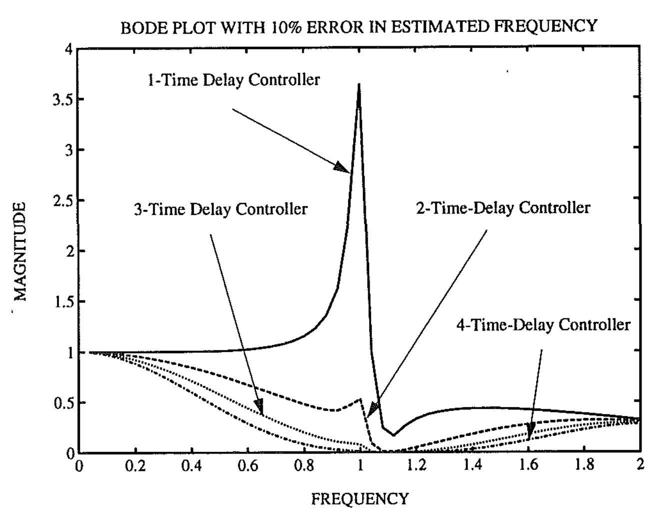

Fig. 8 Magnitude plots of bode diagrams for the time-delay controllers with 10 percent error in estimated frequency

Inputs to Reduce System Vibrations," ASME Journal of Dynamic Systems, Measurement, and Control, Vol. 112, pp. 76–82.

Singh, T., and Heppler, G. R., "Shaped Input for Multimode System," To appear in the ASME Journal of Dynamic Systems, Measurement, and Control.

Singhose, William E., Singer, Neil C., and Seering, W. P., 1990, "Shaping Inputs to Reduce Vibration: A Vector Diagram Approach," *Proceedings of the 1990 IEEE International Conference of Robotics and Automation*, Vol. 2, Cincinnati, Ohio, pp. 922–927.

Tallman, G. H., and Smith, G. H., 1958, "Analog Study of Dead-Beat Posicast Control," *IRE Transactions on Automatic Control*, Vol. AC-3, pp. 14–21.

Wie, Bong, and Liu, Qiang, 1990, "Feedforward/Feedback Control Synthesis for Performance and Robustness," *Proceedings of the 1990 AIAA Guidance, Navigation, and Control Conference*, Portland, OR, Vol. 2, pp. 996–1005.

Experiments on the Tracking Control of a Flexible One-Link Manipulator

Chia-Hsiang Menq[1] and Jack Zhijie Xia[1]

Experimental study on the tracking control of a flexible one-link manipulator is reported in this paper. A tracking control scheme is developed based on the system transfer function. In the proposed control scheme, the desired control input for a given end-point trajectory is obtained by using a command feedforward controller instead of solving the inverse dynamic equations of the system. The proposed control scheme requires small amount of computations and can be easily implemented for real time control. The experimental results are presented, which shows very good tracking performance.

1 Introduction

The control objective of the end-point tracking control of a flexible manipulator is to make the end-point of a flexible arm follow a given trajectory. The main difficulty in the tracking control of a flexible manipulator is due to the fact that the control input is the torque acting on the hub of the flexible link while the feedback signals coming from the end-point. Unlike the rigid robot, for a flexible robot, the dynamics at the actuator is quite different from that at the end-point. It has been shown by many researchers that such a system is a nonminimum phase system. Therefore, the system's inverse dynamic equations are unstable, and the desired torque for a given trajectory is difficult to obtain. The reported approaches for this problem attempted to study the characteristics of this set of unstable differential equations and to obtain the solution of the inverse dynamic equations (Kwon and Book, 1990; Bayo et al., 1988). The instability problem of the system's inverse model is avoided by separating the system into the causal part and the noncausal one (Kwon and Book, 1990) or transferring the dynamic equations to the frequency domain (Bayo et al., 1988). These approaches are proved to be successful based on the simulation and experimental results. However, all of them require computations before actually applying control action, and the off-line implementation is usually required.

In this research, an experimental study on the end-point tracking control of a flexible one-link manipulator is performed. This study serves as a starting point of investigating the end-point tracking control of a general flexible manipulator. A tracking control scheme is developed based on the system's transfer function instead of the system's dynamic equations. Due to the fact that the nonminimum phase characteristics of the system is much more clear by examining the transfer function than by the dynamic equations, the instability problem of the system's inverse model can be easily avoided, and a much simpler tracking control scheme is proposed, which is very efficient and is suitable for real time control.

[1]Department of Mechanical Engineering, The Ohio State University, Columbus, OH 43210.

Contributed by the Dynamic Systems and Control Division of The American Society of Mechanical Engineers. Manuscript received by the Dynamic Systems and Control Division September 1990; revised manuscript received July 28, 1992. Associate Technical Editor: K. Yousef-Taumi.

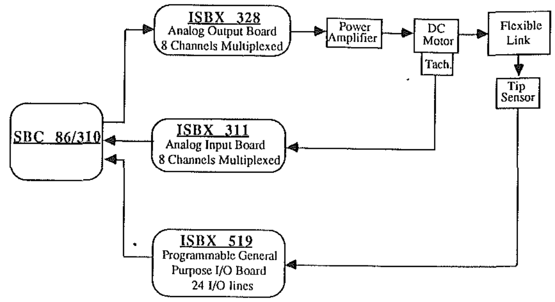

Fig. 1 Schematic diagram of experimental apparatus

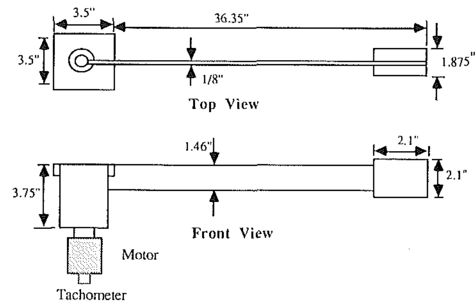

Fig. 2 The flexible link

Table 1 Material properties of the flexible link

Material	6061-T6 aluminum alloy
Young's modules	10.3×10^6 psi
Unit weight	0.098 lb/in.^{3}

2 Experimental Setup

The experimental apparatus, as shown in Fig. 1, consists of a flexible link, an SBC 86/310 computer, expansion I/O boards, interface circuitry, and tip and hub sensors. The flexible link, as shown in Fig. 2, is made of 6061-76 aluminum alloy. The cross section is designed to provide considerable flexibility in the lateral direction while maintaining sufficient stiffness in the vertical direction. The material property of the system is shown in Table 1. Two measurements are available, a tachometer to measure the hub rate, and a linear incremental encoder (HEDS-9200, by Hewlett Packard) on the tip, which has a resolution of 0.00667 inch (0.16 mm), to provide the tip position. The use of the linear incremental encoder provides accurate tip position for tracking control. However, it limits the motion range of the end-point. With a DC torque motor/tachometer unit (SMT3402-U/T from Magnedyne) installed on the hub, the beam structure is able to move on a horizontal plane freely. Since the manipulator is driven directly by the motor, joint compliance is negligible for this experimental apparatus.

It is well known that the joint of an actuator is always subjected to friction force, which is difficult to model. Moreover, the friction force in the actuator will make the system become nonlinear. To compensate the nonlinear friction force, an inner loop is introduced in the system as shown in Fig. 3.