Minimax Robust Jerk Limited Control of Flexible Structures
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Abstract: This paper is concerned with the design of robust jerk limited controllers for flexible structures. In order to eliminate residual vibration at the end of the maneuver, a time-delay filter, forming the command input, is used. A minimax optimization scheme is employed to make the controller more robust with respect to parametric uncertainties. The degree of robustness can be chosen arbitrarily. The minimax setting permits for the easy inclusion of statistical information concerning these parametric uncertainties. The controller design accounts for both limits in the magnitude as well as the time-rate of change of the control input. Among other benefits, this class of control sequences shows a favorable frequency spectrum compared with the jerk unconstrained sequences. The method is illustrated on the undamped as well as the damped Floating Oscillator benchmark problem.

Résumé: Cet article traite la conception de contrôleurs robustes et anti-secousse pour des structures flexibles. Afin d' éliminer les vibrations résiduelles en fin de manœuvre, un filtre à retard de temps, formant la commande, est utilisé. Un arrangement d'optimisation min-max est employé pour rendre le contrôleur moins sensibles aux incertitudes paramétriques. Le degré de robustesse peut être choisi arbitrairement. La formule en problême d'optimisation min-max permet l'inclusion facile d'informations statistiques sur ces incertitudes paramétriques. La conception de ce contrôleur tient compte non seulement des limites dans la taille mais aussi du temps-cadence du changement de commande. En plus d'autres advantages cette classe de commandè montre un spectre de fréquence plus favorable comparé aux classes de commande non limités en secousse.

La méthode s'applique sur le problème de référence, un système de deux-masses-ressort-et-amortisseur, l' oscillateur flottant.

1 Introduction

Extensive research has been undertaken in the field of vibration control of slewing flexible structures. The results obtained have been applied to control problems as diverse as large space structures [1], flexible arm robots [2], computer disk drives [3] and cranes [4]. Most of these control problems demand minimum maneuver time under the condition of quiescent final states. Various constraints have been added to this control problem, such as maximum deflection permitted and limits on the fuel consumed.

In 1957, Smith [5] presented a new approach to the control of flexible structures, which he called "Posicast". The idea behind this control technique is to divide a step input into a number of spaced excitations, which are chosen such that oscillations induced by the earlier steps are eliminated again by later steps. Originally, this method was derived for second order systems having only one input and only one resonant mode. Singer and Seering [6] suggested a practice to design controllers addressing the sensitivity issues of the original controller. In order to make the controller more robust, they equated the derivative of the constraint equation with respect to the natural frequency of the resonant mode to zero. The "Posicast" controller was also extended to cover multi-mode systems.

Singh and Vadali [7] were able to find an equivalent, but frequency-based solution to the problem of designing an appropriate input prefilter which eliminates residual vibration. They developed a pole cancellation formulation where zeros of the time-delay filter are placed atop the oscillatory poles of the system. It was also illustrated that the degree of robustness can be increased by placing multiple zeros of the time delay filter on top of the system poles.

Recently, Singh [8] proposed a minimax formulation to desensitize the input preshaping controller with respect to parametric uncertainties. In this paper, the method is extended to include constraints on the maximum time rate of change of the control input. Most of the design techniques presented so far arrive at bang-bang profiles, which means that the actuator must be able to track infinite jerk control profiles. Jerk is indicative of the time-rate of change of the inertia forces and is thus a measure of the impact levels that can excite unmodeled dynamics. The resulting control profiles are restricted in their maximum time rate of change. This leads to a roll-off of higher frequencies of the control profiles' frequency spectrum. Since the higher frequency dynamics are usually not modeled, the control performance will benefit from such a high frequency roll-off. Other beneficial effects are the reduction of wear on the mechanism, lower acoustical noise and less excitation of the oscillatory modes.

Some authors have already considered reducing the time rate of change of the control profile. Bhat and Miu [3] propose to use the time integral of the square of the absolute instantaneous jerk as a cost function for the controller design. Hindle and Singh [9] reformulated this problem as the minimization of a weighted combination of jerk and power consumed. They also proposed to use the state sensitivity equation to arrive at control profiles which are insensitive to modelling errors. Both of these approaches do not account for finite limits on the time-rate of change of the control profile.

In the presented paper, the switching times are not limited to integer multiples of a fixed sampling time, but can be allocated freely. This provides a globally optimal solution and can be used as a benchmark to compare ap-

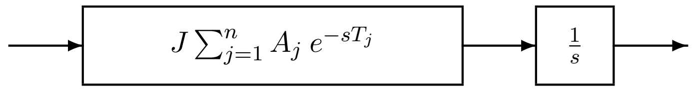

Figure 1: Structure of the Time Delay Filter

proximating techniques. Robustness can be added as desired. The paper consists of three major sections. In the first section, the problem is outlined and the optimization problem is formulated. Then, the Van Loan identity and the pseudo-residual energy approach are briefly reviewed. They are reformulated and applied to the design of jerk restricted controllers. The pseudo-energy approach is elucidated for both undamped as well as damped systems. Numerical simulations conclude this paper and are used to illustrate the potential of the proposed technique. All examples are presented for the Floating Oscillator, which serves as a benchmark, allowing to compare different controllers on the same problem [10].

2 Problem Statement

This paper is concerned with flexible structures, which can be described by the set of second order differential equations

$$M\,\ddot{y}(t) + C(p)\,\dot{y}(t) + K(p)\,y(t) = D\,u(t), \quad (1)$$

where M is the mass matrix, C the (optional) damping matrix, K the stiffness matrix and D the input distribution matrix, which defines how the input $u(t)$ is acting on the different system states $y(t)$. For single input systems, D is the input distribution vector. Parametric uncertainty is introduced into the system by means of the uncertain vector p, known to be bounded by

$$p_L \le p \le p_U. \quad (2)$$

The mass matrix is assumed to be known precisely, because mass can usually be estimated very accurately and is the least affected by aging or wear-out.

Good control performance necessitates extraction of residual vibration. The pseudo-residual energy function is chosen as a measure for the amount of remanent vibration, thus

$$\begin{aligned} F =& \frac{1}{2}\dot{y}^T M \dot{y} + \frac{1}{2} y^T K y \\ &+ \frac{1}{2}(y - y_{final})^T K_{hyp}(y - y_{final}) \end{aligned} \quad (3)$$

evaluated at $t = T_{Eval}$. The last term must be introduced to account for rigid body dynamics. This term can be interpreted as the presence of a hypothetical spring. This approach was described in detail by Singh [8]. For this cost function, analytical gradients can easily be obtained. This allows to expedite the convergence of the optimization package which is used to obtain the controller parameters.

As was stated in the introduction, constraints will be imposed which limit both, the magnitude as well as the time-rate of change of the control input,

$$\dot{u}(t) \le J \quad (4)$$

$$u(t) \le 1 \quad (5)$$

The system will be controlled by a time-delay filter structure as shown in Fig. 1. To maneuver the structure in minimal time under the constraints imposed, the input $u(t)$ will consist of a sum of delayed ramp signals,

$$u(t) = J \sum_{j=1}^{n} A_j \langle t - T_j \rangle, \quad (6)$$

where $\langle x \rangle = x\mathcal{H}(x)$ and $\mathcal{H}(x)$ denotes the Heavyside function. Also, $T_n = T_{final}$, and A_j is restricted to be

$$A_j \in \{-2,\ -1,\ 1,\ 2\}\ \forall\ j. \tag{7}$$

For the first switch, i. e. $j = 1$, $A_1 = 1$ and $T_1 = 0$. Similarly, for the last switch, i. e. $j = n$, $A_n \in \{-1,\ 1\}$.

In order to accurately calculate the response of the time-delay filter, the Van Loan identity [11] will be used. Upon rewriting the system in first order state-space form,

$$\dot{x}(t) = Q\,x(t) + R\,u(t) \tag{8}$$
$$y(t) = S\,x(t) + T\,u(t), \tag{9}$$

the matrix exponential can be utilized to calculate the unit step response of this linear systems. The matrix

$$P = \begin{bmatrix} Q & R \\ 0 & 0 \end{bmatrix} \tag{10}$$

has to be constructed. P is the concattenation of Q and R and is padded with one row of zeros. The Van Loan identity states that

$$M = e^{PT} = \begin{bmatrix} e^{QT} & \int_0^T e^{Q(T-\tau)} R\ d\tau \\ 0 & 1 \end{bmatrix}. \tag{11}$$

Thus, one can easily and accurately calculate the convolution integral, which delivers the step responses of the states of an LTI system for any arbitrary time T. This approach yields more precise results than solving the system of ordinary differential equations by means of a numerical integration technique. Numerical precision becomes important as the differences between consecutive switches might get very small for certain regions of jerk. Since every jerk limited time-optimal control profile will consist of a sum of delayed ramp inputs and not of a sum of delayed step inputs, the system has to be augmented by an integrator in the input path.

Minimax Robustification: As the maximum residual vibration over the prescribed uncertain space has to be minimized, the problem statement is formulated as

$$\min_X\ \max_p\ F(X, p), \tag{12}$$

where X is the vector of design variables used to describe the control profile, p is a point in the uncertain space, and F is the cost function. The cost function is evaluated for $t = T_{final}$, since the residual vibration at and after the final time should be minimal.

A constraint can be added to ensure that the residual energy is zero at the nominal values of all system parameters. This is stated as

$$g(X) = F(X, p)|_{p=p_{nom}} = 0. \tag{13}$$

This constraint puts more emphasis on the nominal point, enhancing control performance in the vicinity of the nominal parameter set. Usually, the system parameters are more likely to lie close to p_{nom}, thus this parameter set should deserve special consideration.

3 Numerical Examples

This section will show examples for the single- and multi-parameter desensitization. The first system under investigation is the undamped Floating Oscillator. Desensitization with respect to the spring stiffness is carried out. In addition, robustified control sequences for the damped Floating Oscillator will be designed later in this section. The damped Floating Oscillator has two uncertain system parameters, the spring stiffness k and the damping coefficient c, which leads to a two-dimensional uncertain space.

Floating Oscillator without Damping: The outlined robustification method was applied to the Floating Oscillator benchmark problem, which is shown in Fig. 2.

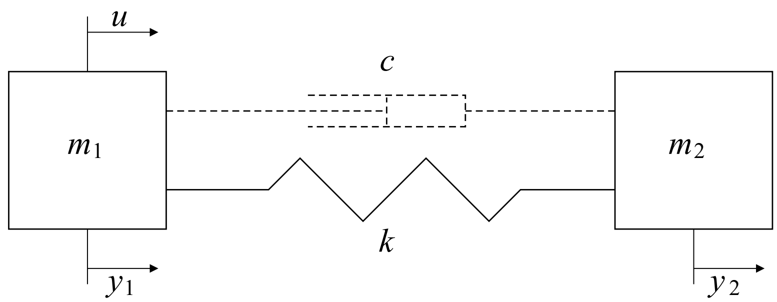

Figure 2: Floating Oscillator Benchmark Problem

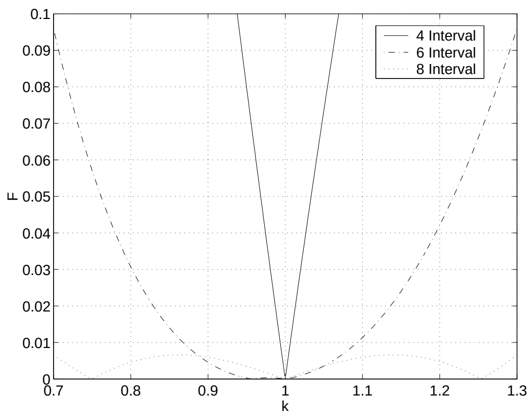

Figure 3: Cost for $J = 1.6$, Constrained Case

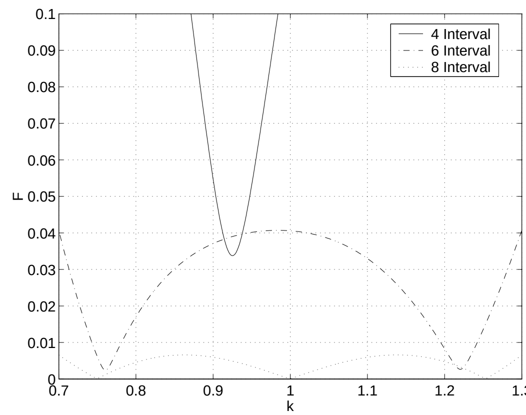

Figure 4: Cost for $J = 1.6$, Unconstrained Case

This system is described by

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \end{bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix} \begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \tag{14}$$

The system parameters m_1 and m_2 are unit masses. The spring stiffness k is assumed to lie in the interval $0.7 \leq k \leq 1.3$ with a nominal value of $k_{nom} = 1$. For the undamped Floating Oscillator considered first, c is set to zero. For the examples with added damping, the damping coefficient is assumed to be from the interval $0 \leq c \leq 0.2$. Here, the nominal value will be $c_{nom} = 0.1$.

Profiles for the Undamped Oscillator: For the examples in this section, a maximum allowable jerk of $J = 1.6$ has been selected. Both the constrained and the unconstrained case have been investigated. First, the results for the optimization with the imposed equality constraint are presented. In order to see the influence of the number of intervals on the cost and the final time, control profiles for four, six and eight intervals have been calculated. In Fig. 3, the value of the objective function has been plotted over the uncertain parameter k. The larger the value, the more vibratory energy is stored in the system at the end of the maneuver. In the adjacent figure, Fig. 4, cost curves obtained without the requirement of zero cost at k_{nom} have been

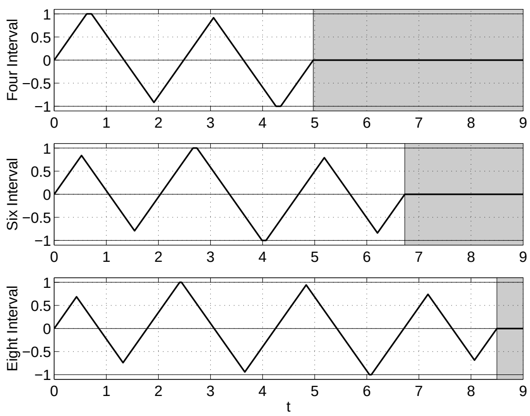

Figure 5: Control Profiles for $J = 1.6$, Constrained Case

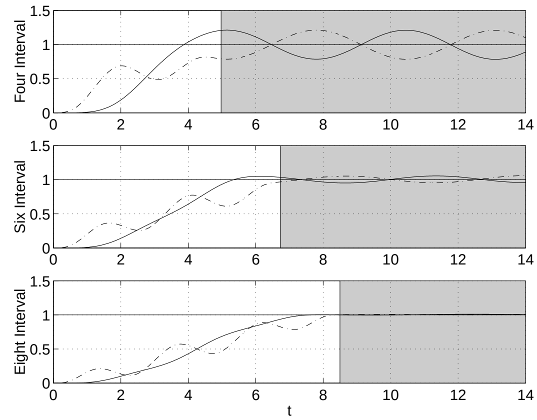

Figure 6: Response for $k = 0.7$ (Non-Nominal), Constrained Case

graphed. One can see how the maximum cost over the entire interval could be reduced by abolishing this constraints. This comparison also illustrates that the control performance near the nominal parameter set can degrade drastically if this constraint on zero residual energy is given up. The eight interval profile is the only control sequence for which the residual energy remains close to zero for the nominal parameter set regardless of enforcing constraint Eq. 13. The control profiles found as a solution to the constrained optimization problem have been plotted in Fig. 5. One can clearly see the ramping, jerk limited structure of the profile. The end of the maneuver is marked with a vertical line. Time after the end of the control sequence is shaded grey.

In order to illustrate the added robustness, the system response for a spring stiffness of $k = 0.7$ is graphed in Fig. 6. In this diagram, it can be seen, how maneuver time is traded for robustness. While the four interval profile is the shortest and would thus maneuver the nominal system in the shortest amount of time and still warrant quiescent states, its control performance is the worst for the perturbed system. Here, the more robust profiles excite less vibration.

Another interesting diagram is the development of the switching times as a function of the maximum permissible jerk as shown in Fig. 7. Here, one can see the trajectories of the switching times as a function of the maximum permissible jerk in the interval $0.1 \leq J \leq 1000$. The thick vertical lines mark points, where some of the switches collapse. The term "collapse of switches" refers to the fact that two switches of the same amplitude occur at the same instant in time and can thus be replaced by one switch with twice the amplitude of the individual switches. The small boxes inserted into the diagram show the shape of the control profile for certain regions of jerk. Small vertical arrows illustrate the correlation between the trajectories of the switching times and the control profile. From this diagram, one can also read out the maneuver time which is represented by the topmost line in the diagram. The time rate of change can be limited extensively without significantly increasing the maneuver time.

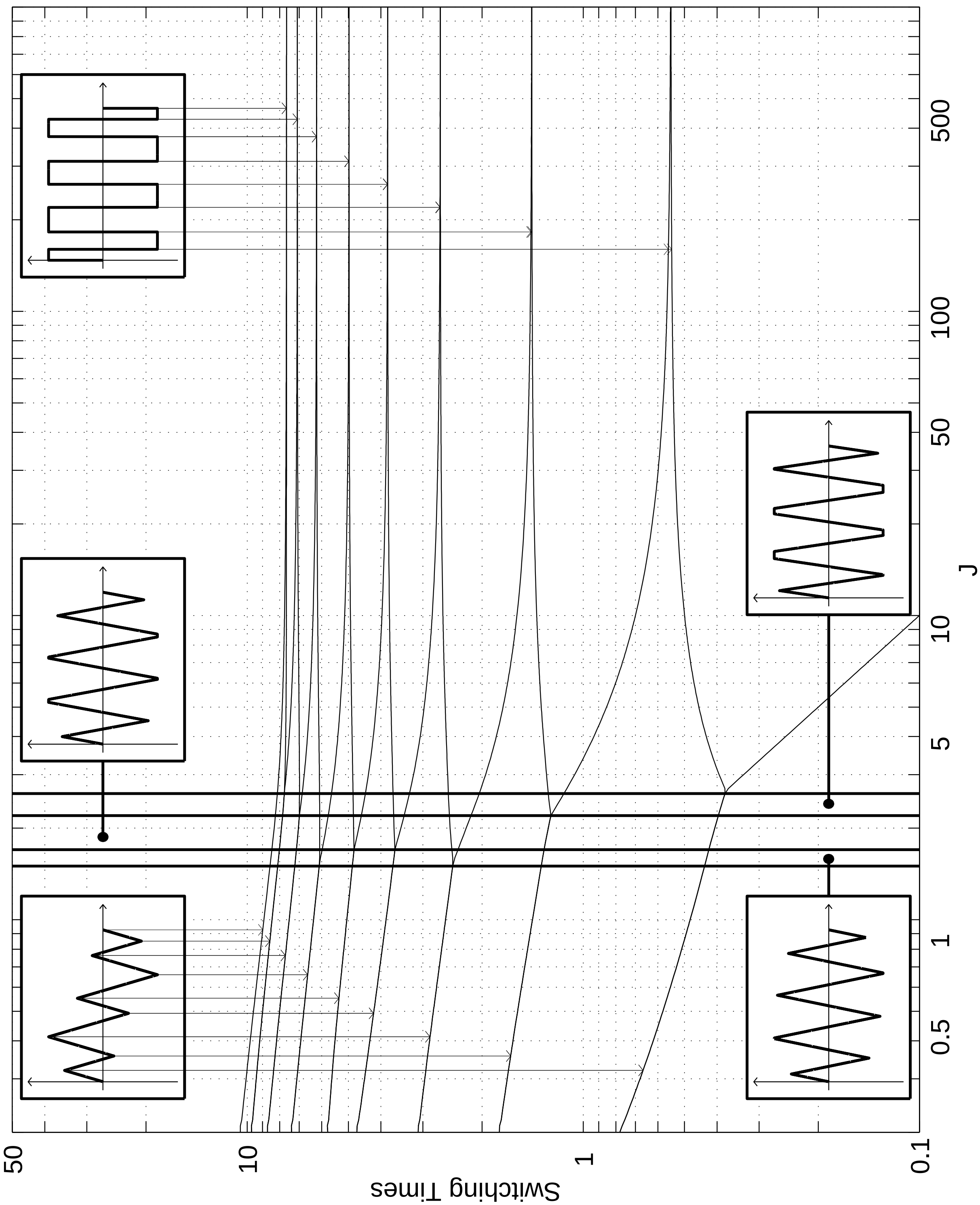

Figure 7: Trajectories for Switching Times

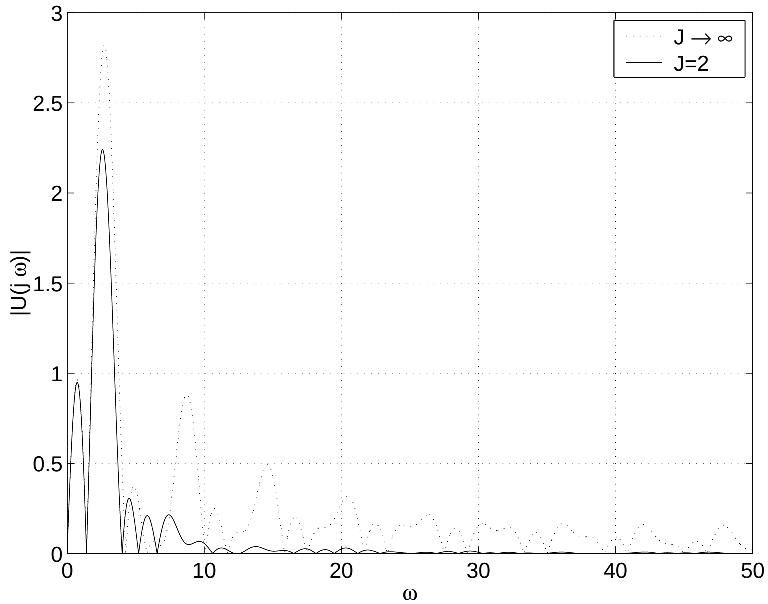

Figure 8: Comparison of Frequency Spectra

For example, upon reducing the jerk from $J = 1000$ to $J = 1$, the maneuver time, T_{final}, increases from $T_{Final} = 7.6397$ to $T_{Final} = 8.9163$. Finally, one can see a comparison of the power spectrum of a a jerk unlimited control profile and a jerk limited control profile in Fig. 8. This diagram illustrates, how the higher frequency content has been reduced by limiting the time-rate of change of the control profile.

Floating Oscillator with Damping: In the following, the robustification method is applied to the damped Floating Oscillator, depicted in Fig. 2. For this class of systems, the control profile can no longer be forced to be symmetric about the mid-maneuver time. For all examples, the jerk is limited to be $J = 2$. The number of intervals is varied from four to eight. For the examples presented in this paper, the constraint shown in Eq. 13 is enforced. This results in zero cost for the point $k = k_{nom}$ and $c = c_{nom}$ on the cost surface. The different cost surfaces for the constrained case are shown in Fig. 9.

For the unconstrained case, similar results have been obtained. Due to this similarity, a separate figure has not been included.

4 Conclusions

The proposed robustification technique has been implemented for two benchmark problems, thus demonstrating the feasibility of the minimax approach for both one- and two-dimensional uncertain spaces. The extension to higher dimensional spaces is straightforward, therefore no such examples have been included in this paper. For a system where all parameters are known or at least expected to be close to their nominal values, it is reasonable to force the cost function to zero at the nominal set of parameter values. However, it has been shown that the maximum cost over the prescribed uncertain space can be reduced by taking this constraint out of the optimization statement. It has been evidenced that the number of intervals in the control profile influences the sensitivity. By increasing the number of switches, the control

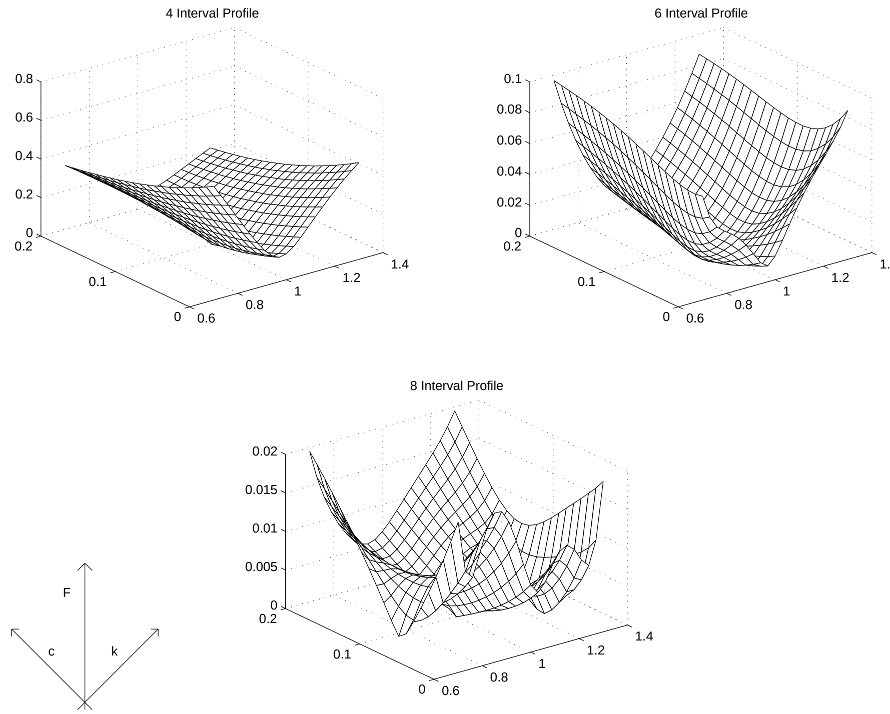

Figure 9: Cost Surfaces for Uncertainties in both c and k, Constrained Case

insensitivty to the uncertain parameters has been augmented. The final time increased with the number of switches. This proves that robustness and response time are competing design goals. The extension to multi-input systems is straightforward and has been presented in detail in [12].

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